Monopoly Example

$p(q)$

$$\pi(q) = p(q)\,q - c(q)$$

Demand function $q(p) = 100 - p$. Cost function for the firm: $c(q) = 10q$

The inverse demand is: $p(q) = 100 - q$

$$\pi(q) = (100 - q)\,q - 10q$$

$$\begin{pmatrix} 10 & 90 & 800 \\ 20 & 80 & 1400 \\ 30 & 70 & 1800 \\ 40 & 60 & 2000 \\ 45 & 55 & 2025 \\ 50 & 50 & 2000 \\ 60 & 40 & 1800 \\ 70 & 30 & 1400 \\ 80 & 20 & 800 \\ 90 & 10 & 0 \end{pmatrix}$$

$$\frac{\partial((100 - q)\,q - 10q)}{\partial q} = \frac{\partial\left(100q - q^2 - 10q\right)}{\partial q} = (100 - 2q) - 10 = 90 - 2q$$

$$(100 - 2q) = 10$$

$$MR = MC$$

$$90 = 2q$$

$$q = 45, p = 55, \pi = 2025$$

Markup and Elasticity

The markup equation relates consumer elasticity at the optimum to the firm's marginal cost.

$$p(q)\,q - c(q)$$

At the optimum, the firm sets marginal profit to zero:

$$\frac{\partial\left(p(q)\,q - c(q)\right)}{\partial q} = 0$$

Using the product rule on $p(q)\,q$ we get:

$$\frac{\partial p}{\partial q} q + p = mc$$

Elasticity ε is:

$$\varepsilon = \frac{\partial q}{\partial p}\frac{p}{q}, \frac{1}{\varepsilon} = \frac{\partial p}{\partial q}\frac{q}{p}$$

Notice the term $\frac{\partial p}{\partial q} q$ is almost $\frac{1}{\varepsilon}$. Let's divide both sides by p

$$\frac{\partial p}{\partial q}\frac{q}{p} + 1 = \frac{mc}{p}$$

$$\frac{1}{\varepsilon} + 1 = \frac{mc}{p}$$

$$\frac{1+\varepsilon}{\varepsilon} = \frac{mc}{p}$$

$$\frac{p}{mc} = \frac{\varepsilon}{1+\varepsilon}$$

The markup is $\frac{p}{mc}$. The markup is how much the firm will charge in excess of the cost to produce.

In our example $mc = 10$. The price the firm charges is $p = 55$.

$$\frac{p}{mc} = \frac{55}{10} = 5.5$$

Let's calculate $\frac{\varepsilon}{1+\varepsilon}$. $q(p) = 100 - p$.

$$\varepsilon = \frac{\partial (100 - p)}{\partial p} \frac{p}{100 - p}$$

$$= -\frac{p}{100 - p}$$

$$= -\frac{55}{100 - 55} = -\frac{11}{9}$$

We can calculate the markup:

$$\frac{-\frac{11}{9}}{1 + \left(-\frac{11}{9}\right)} = \frac{11}{2}$$

Suppose we know a firm charges $p = 10$. And we know the elasticity of demand is -2. From this, we can back out marginal cost:

$$\frac{p}{mc} = \frac{-2}{1 + (-2)}$$

$$\frac{10}{mc} = \frac{-2}{1 + (-2)}$$

$$\frac{10}{mc} = 2$$

$$mc = 5$$

How do we know the monopolist will actually mark **up price** over their marginal cost?

Proposition: If demand is elastic, markup will be > 1. $p > mc$.

Using the markup equation, $\frac{p}{mc} = \frac{\varepsilon}{1+\varepsilon}$. We need to check when $\frac{\varepsilon}{1+\varepsilon} > 1$

$$\frac{\varepsilon}{1+\varepsilon} > 1$$

If demand is elastic, $1 + \varepsilon < 0$

$$\varepsilon < 1 + \varepsilon$$

$$1 > 0$$

We have proved that when demand is elastic that the optimum, the firm will charge more than their marginal cost.

It will also be true that if demand is inelastic at the optimum, the firm will charge less than their marginal cost. **But this will never happen. Why?**

Monopolist Profit is Never Optimal when Demand is Inelastic

At the profit maximizing quantity, demand cannot be inelastic.

Assume demand is inelastic.

$$pq - c\left(q\right)$$

What happens to profit if the firm decreases quantity by 1%

What happens to costs? It has to go down?

What happens to revenue?

If demand is inelastic, then a 1% decrease is quantity is associated with a more than 1% increase in price. Price goes up my more than 1%, demand goes down by 1%. This has to increase revenue.

Any time demand is inelastic, the firm can increase revenue by decreasing quantity.

In summary, when demand is inelastic, decreasing quantity **increases revenue and decreases cost.** So profit has to go up.

So operating where demand is inelastic can never be optimal.

$$q\left(p\right) = \left(\frac{100}{p}\right)$$

This is a demand function that is never elastic, it is always elasticity of -1.

$$p = \left(\frac{100}{q}\right)$$

$$\pi\left(q\right) = \frac{100}{q}q - 10q$$

$$\pi\left(q\right) = 100 - 10q$$

If demand is always inelastic, then the firm will want to drive quantity down to zero.

A monopolists general behavior is to create artificial scarcity to drive up price and increase profit.

Consumer Surplus Under Monopoly

Monopoly Behavior

Types of Price Discrimination

1st Degree

Price	# Buyers	Profit
\$3	1	
\$2	2	
\$1	3	

2nd Degree

3rd Degree

$q_s = 100 - 2p$, $q_n = 100 - p$.

Bundling

	Shirt	Pants	Both
Consumer 1	50	30	80
Consumer 2	10	80	90