Econ 3100 Final Exam,
Spring 2025
(100 points) Name: ______________________________

Write your answers in the space provided. Show your work for quantitative questions.

1. Fill in the blanks:

   (a) (4 points) If a firm's production function is such that doubling output will always require increasing the inputs by more than double, its production function is said to exhibit ____________________.

   Solution: decreasing returns to scale

   (b) (4 points) If demand is ____________________, a monopolist can always increase its revenue **and** lower its cost (thus increasing profit) by decreasing the quantity it sells and increasing the price accordingly.

   Solution: inelastic

   (c) (4 points) A cellphone company offers an unlimited plan with unlimited data, talk, texting, and hotspot service for a single price but does not offer these services individually. This is an example of price discrimination known as ____________________.

   Solution: bundling

   (d) (4 points) The First Welfare Theorem states that under certain conditions the equilibrium of an exchange economy will always be ____________________.

   Solution: Pareto Efficient

   (e) (4 points) Suppose a consumer's demand for good 1 is given by

   $$x_1(p_1, m) = \frac{1}{2}\frac{m}{p_1},$$

   where p_1 is the price of good 1 and m is the consumer's income. The price elasticity of demand for good 1 is ____________________.

   Solution:

   $$\varepsilon_{x_1,p_1} = \frac{\partial x_1}{\partial p_1} \cdot \frac{p_1}{x_1} = \left(-\frac{1}{2}\frac{m}{p_1^2}\right) \cdot \frac{p_1}{\frac{1}{2}\frac{m}{p_1}} = -1.$$

2. Suppose market demand is $q_d = 300 - 2p$ and market supply is $q_s = 2p$.

   (a) (4 points) Find the equilibrium price p and quantity q.

   Solution: $p^* = 75, q^* = 150$

   (b) (4 points) Compute the price elasticity of demand at the equilibrium price.

   Solution: -1

   (c) (4 points) Interpret this elasticity. What does it mean, roughly, about the relationship between quantity demanded and price?

   Solution: falls by 1%

   (d) (4 points) Suppose the government imposes a quantity tax of $t = 50$ per unit. Find the equilibrium price and quantity. How much do consumers pay per unit?

   Solution: $p = 50, q = 100, 100$

   (e) (4 points) Calculate the deadweight loss resulting from this tax.

   Solution: $DWL = 1250$

3. Suppose demand is given by $250 - \frac{1}{2}p$, and firms have cost functions $c(q) = 200q$.

   (a) (4 points) Find the inverse demand.

   Solution: $p = 500 - 2q$

   (b) (4 points) Suppose there is a monopolist in this market. Write down the monopolist's profit function.

   Solution: $\pi(Q) = (500 - 2q)q - 200q = 300q - 2q^2$

   (c) (4 points) Find the profit-maximizing quantity q and corresponding price p for this monopolist.

   Solution: $q = 75$, $p = 350$

   (d) (4 points) Now suppose two firms compete in Cournot competition and both have the same cost function above. Write Firm 1's profit function in terms of q_1 and q_2.

   Solution: $\pi_1 = (500 - 2(q_1 + q_2))q_1 - 200q_1$

   (e) (4 points) Derive firm 1's best-response function.

   Solution: $q_1 = 75 - \frac{1}{2}q_2$

   (f) (4 points) Leverage symmetry to find the Nash equilibrium quantities.

   Solution: $q_1^* = q_2^* = 50$

   (g) (4 points) What is the equilibrium market price p with these two firms?

   Solution: $p^* = 300$, $Q^* = 100$

4. A firm has production function $f(x_1, x_2) = \min\{2x_1, 3x_2\}$. Input prices are $w_1 = \$6$ and $w_2 = \$9$.

   (a) (4 points) What is the slope of this firm's **isocost** curves?

   Solution: $-\frac{2}{3}$

   (b) (4 points) Derive the conditional factor demands for x_1 and x_2 as a function of output y.

   Solution: $x_1(y) = y/2$, $x_2(y) = y/3$

   (c) (4 points) Derive the cost function $c(y)$.

   Solution: $c(y) = 6(y/2) + 9(y/3) = 6y$

   (d) (4 points) What is another input bundle on the isoquant that contains $(x_1, x_2) = (4, 1)$?

   Solution: $(x_1, x_2) = (5, 1)$

5. Consumer A and Consumer B consume goods 1 and 2. Consumer A has endowment $\omega_{1,a} = 15$ and $\omega_{2,a} = 5$. Consumer B has endowment $\omega_{1,b} = 5$ and $\omega_{2,b} = 15$.

   Consumer A has demand:

$$x_{1,a} = \frac{(p_1\omega_{1,a} + p_2\omega_{2,a})}{p_1}, \quad x_{2,a} = 0.$$

   Consumer B has demand:

$$x_{1,b} = \frac{\frac{1}{2}(p_1\omega_{1,b} + p_2\omega_{2,b})}{p_1}, \quad x_{2,b} = \frac{\frac{1}{2}(p_1\omega_{1,b} + p_2\omega_{2,b})}{p_2}.$$

   (a) (4 points) What are the consumer's demands if $p_1 = 1$ and $p_2 = 1$?

   Solution: For consumer A, income is $1 \cdot 15 + 1 \cdot 5 = 20$, so

$$x_{1,a} = \frac{20}{1} = 20, \quad x_{2,a} = 0.$$

   For consumer B, income is $1 \cdot 5 + 1 \cdot 15 = 20$, so

$$x_{1,b} = \frac{\frac{1}{2} \cdot 20}{1} = 10, \quad x_{2,b} = \frac{\frac{1}{2} \cdot 20}{1} = 10.$$

   (b) (4 points) Write down the market-clearing conditions.

   Solution: Total endowments are $\omega_1 = 15 + 5 = 20$ and $\omega_2 = 5 + 15 = 20$. Market clearing requires:

$$x_{1,a} + x_{1,b} = \omega_{1,a} + \omega_{1,b} = 20, \qquad x_{2,a} + x_{2,b} = \omega_{2,a} + \omega_{2,b} = 20.$$

(c) (4 points) Assume $p_1 = 1$. What must p_2 be in equilibrium?

Solution: With $p_1 = 1$, Walras' law implies that if one market clears, so does the other, so it is enough to impose market clearing for a single good. It is easier to use the market for good 2.

Demands for good 2 are

$$x_{2,a} = 0, \qquad x_{2,b} = \frac{\frac{1}{2}(1 \cdot 5 + p_2 \cdot 15)}{p_2} = \frac{5 + 15p_2}{2p_2}.$$

Market clearing for good 2 requires

$$x_{2,a} + x_{2,b} = 20 \;\Rightarrow\; \frac{5 + 15p_2}{2p_2} = 20.$$

Thus

$$5 + 15p_2 = 40p_2 \;\Rightarrow\; 5 = 25p_2 \;\Rightarrow\; p_2 = \frac{1}{5}.$$

(d) (4 points) Find the consumers' equilibrium *allocations* by determining what they demand at these prices.

Solution: At $(p_1, p_2) = (1, \frac{1}{5})$:

For consumer A,

$$m_a = 1 \cdot 15 + \tfrac{1}{5} \cdot 5 = 16, \quad x_{1,a} = \frac{16}{1} = 16, \quad x_{2,a} = 0.$$

For consumer B,

$$m_b = 1 \cdot 5 + \tfrac{1}{5} \cdot 15 = 8,$$

$$x_{1,b} = \frac{\frac{1}{2} \cdot 8}{1} = 4, \quad x_{2,b} = \frac{\frac{1}{2} \cdot 8}{\frac{1}{5}} = \frac{4}{1/5} = 20.$$

Thus the equilibrium allocations are

$$(x_{1,a}, x_{2,a}) = (16, 0), \qquad (x_{1,b}, x_{2,b}) = (4, 20).$$