

EXERCISES CHAPTER 4

Exercise 1. Consider bundles a , b , and c with the given utilities $U(a) = 8$, $U(b) = 15$, and $U(c) = 10$. What complete and transitive preference relation does this utility function represent (written in chain notation)?

Exercise 2. Provide an alternative utility function that represents the same preferences as those in the previous exercise.

Exercise 3. Suppose that a consumer's preferences can be represented by the utility function $u(x_1, x_2) = \sqrt{x_1} + x_2$. Which is true of this consumer's preferences?

$$(16, 3) \succ (4, 5), (4, 5) \succ (16, 3), \text{ or } (4, 5) \sim (16, 3)$$

Exercise 4. Suppose that a consumer's preferences can be represented by the utility function $u(x_1, x_2) = x_1 x_2$. Which is true of this consumer's preferences?

$$(8, 2) \succ (4, 4), (4, 4) \succ (8, 2), \text{ or } (4, 4) \sim (8, 2)$$

Exercise 5. For the set $X = \{p, q, r\}$, write down a utility function that represents each of these preference relations.

- (1) $p \succ q, q \succ r, r \succ p, p \succ q, q \succ r, p \succ r$
- (2) $p \succ q, q \succ r, r \succ p, p \succ q, q \succ p, q \succ r, r \succ q, p \succ r, r \succ p$

Exercise 6. Suppose that a consumer's preferences can be represented by the utility function $u(x_1, x_2) = \sqrt{x_1} + x_2$. They currently have the bundle $(9, 4)$. What is the smallest x_2 the consumer would accept (with zero x_1) in exchange for the bundle $(9, 4)$?

Exercise 7. Suppose that a consumer's preferences can be represented by the utility function $u(x_1, x_2) = x_1 x_2$. They currently have the bundle $(9, 4)$. What is the smallest amount x^* such that the consumer would accept the bundle (x^*, x^*) in exchange for the bundle $(9, 4)$?

Exercise 8. For each of the following utility functions, find the MRS and use this to calculate the slope of the consumer's indifference curve at $(2, 2)$.

- (1) $u(x_1, x_2) = 3x_1 + 2x_2$
- (2) $u(x_1, x_2) = x_1x_2$
- (3) $u(x_1, x_2) = 4x_1x_2 + 10$
- (4) $u(x_1, x_2) = \ln(x_1) + x_2$
- (5) $u(x_1, x_2) = x_1 + x_1x_2$

Exercise 9. Of the utility functions above, which represent the same preferences as each other?

Exercise 10. Sketch a few indifference curves of the following utility functions.

- (1) $u(x_1, x_2) = 2x_1 + 3x_2$
- (2) $u(x_1, x_2) = \min\{2x_1, x_2\}$
- (3) $u(x_1, x_2) = \max\{x_1, x_2\}$

Exercise 11. Suppose a consumer is buying a car. A car is represented by a bundle (c, h) where c is cupholders and h is horsepower. Unlike usually, c can only be an integer and is always one of these numbers $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Horsepower can be any number including non-integers but is always between 0 and 1000. That is, in the interval $[0, 1000]$. A consumer has “lexicographic” preferences over cars, prioritizing cup holders over horsepower. For two cars, if one has more cup holder, they like it more. If they have the same number of cup holders, they like the one with more horsepower better. Write down a utility function that represents these preferences.