EXERCISES CHAPTER 6

Exercise 1. Suppose that prices are $p_1 = 1, p_2 = 2$ and income is $m = 60$. Utility is $u(x_1, x_2) = x_1 + x_2$.

a) What is the slope of this consumer's indifference curves? Interpret this number.
b) What is the slope of their budget line? Interpret this number.
c) Given these interpretaions, describe why their optimal bundle involves consuming only one of the two goods.
d) What is their optimal bundle?

Exercise 2. Suppose that prices are $p_1 = 5, p_2 = 2$ and income is $m = 60$. Their utility $u(x_1, x_2) = 2x_1 + 5x_2$.

a) What is this consumer's utility of spending all their money on good one?
b) What is their utility of spending all their money on good two?
c) What utility do they get if they spend half of their money on each good?

Exercise 3. Suppose that prices are $p_1 = 1, p_2 = 2$ and income is $m = 60$. Their utility is $u(x_1, x_2) = min\{x_1, x_2\}$.

a) Write down the "no waste condition" for this consumer.
b) Write down the equation for the budget line for this consumer.
c) What is their optimal bundle?

Exercise 4. Suppose that prices are $p_1 = 1, p_2 = 2$ and income is $m = 60$. Their utility function is $u(x_1, x_2) = max\{x_1, x_2\}$. Find their optimal bundle. Use your intuition based on what you know about these preferences from studying them in previous problems.

Exercise 5. Suppose that prices are $p_1 = 1, p_2 = 2$ and income is $m = 60$. Their utility is $u(x_1, x_2) = x_1 x_2$.

a) What is the slope of the consumer's indifference curves (MRS)?
b) What is the slope of their budget line?
c) Write an equation that implies that their indifference curve is tanget to their budget line.
d) What bundle on their budget line meets the condition above? That is, it solves their budget equation **and** the condition above.
e) How does the utility of this bundle compare to the utility of buying all x_1 or all x_2?
f) What is their optimal bundle?

Exercise 6. A rectangle has area $w * h$ and perimeter $2w + 2h$. Suppose you have to draw a rectangle that has perimeter 20.

a) Write the perimeter constraint as a budget constraint.
b) What w, h maximize the area?

Exercise 7. Suppose that prices are $p_1 = 1, p_2 = 2$ and income is $m = 60$. Find the optimal bundle if $u(x_1, x_2) = x_1^2 + x_2^2$.

a) What is the slope of the consumer's indifference curves (MRS)?
b) What is the slope of their budget line?
c) Write an equation that implies that their indifference curve at a bundle is tanget to their budget line.
d) What bundle on their budget line meets the condition above? That is, it solves their budget equation **and** the condition above.
e) How does the utility of this bundle compare to the utility of buying all x_1 or all x_2?
f) What is their optimal bundle?

Exercise 8. A consumer makes pies. x_1 is apples and x_2 is crusts. It requires 2 apples and 1 crust to make a pie. They only care about the number of pies they make so their preferences are represented by $min\left\{\frac{1}{2}x_1, x_2\right\}$. Their income is $m = 60$.

a) If prices are $p_1 = 1$, $p_2 = 2$. What is their optimal bundle?
b) The grocery store starts a promotion. You get 10 apples for free if you buy two crusts. Sketch their budget set.
c) What is their optimal bundle? Add the indifference curve that includes the optimal bundle on your sketch above.

Exercise 9. A consumer's utility function is $ln\left(x_1\right) + ln\left(x_2\right) + ln\left(x_3\right)$. Prices are $p_1 = 1, p_2 = 1, p_3 = 2$. Income is $m = 60$.

a) Set up their Lagrangian. Remember to think of a version of their utility function that is penalized by λ for every dollar they spend more than their income.
b) Find the first order conditions where the partial derivative of the lagrangian is 0 for x_1, x_2, x_3, λ.
c) Solve these equations to get the optimal bundle.

Try these at home.

Exercise 10. Suppose that prices are $p_1 = 1, p_2 = 2$ and income is $m = 60$. Their utility is $u(x_1, x_2) = 2x_1 + 5x_2$.

a) What is this consumer's utility of spending all their money on good one?
b) What is their utility of spending all their money on good two?
c) What utility do they get if they spend half of their money on each good?
d) What is their optimal bundle?

Exercise 11. A consumer's utility function $min\left\{x_1, x_2, x_3\right\}$. Prices are $p_1 = 1, p_2 = 1, p_2 = 2$. Income is $m = 60$. What is their optimal bundle?

Exercise 12. A consumer's utility function $max\left\{x_1, x_2, x_3\right\}$. Prices are $p_1 = 1, p_2 = 2, p_2 = 3$. Income is $m = 60$. What is their optimal bundle?

Exercise 13. A consumer's utility function is $min\left\{x_1, \frac{1}{3}x_2\right\}$. Prices are $p_1 = 1, p_2 = 1$. Income is $m = 60$. What is their optimal bundle?

Exercise 14. A consumer's utility function is $x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$. Prices are $p_1 = 1, p_2 = 1$. Income is $m = 60$. What is their optimal bundle?