

1 Budget

$(2, 2)$ bundle 2 units of x_1 and 2 of x_2 .

Budget set is the set of all affordable bundles (x_1, x_2)

$$p_1x_1 + p_2x_2 \leq m$$

Budget line is the set of all bundles that cost exactly income:

$$p_1x_1 + p_2x_2 = m$$

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$$

Slope (**how much x_2 you have to give up to get one more unit of x_1**).

$$-\frac{p_1}{p_2}$$

Intercepts of the budget line:

$$\left(\frac{m}{p_1}, 0\right), \left(0, \frac{m}{p_2}\right)$$

2 Relations and Preference Relation

A relation is a set of statements about **pairs** of things in a set.

$\{x, y, z\}$

Reflexive: Everything is related to itself.

$$xRx, yRy, zRz$$

Complete: Every **pair** of things (including a thing and itself) have some statement that is true. A complete relation must be reflexive.

This is complete:

$$xRx, yRy, zRz, xRy, yRz, zRx$$

This is not complete because x and z have no relationship.

$$xRx, yRy, zRz, xRy, yRz$$

This is not complete because it is not reflexive

$$yRy, zRz, xRy, yRz, zRx$$

Transitive: If aRb, bRc then it must be aRc is also true.

This is not transitive:

$$xRx, yRy, zRz, xRy, yRz, zRx$$

This is transitive:

$$xRx, yRy, zRz, xRy, yRz, xRz$$

3 Preference Relations

$\succsim$ “weak” preference relation. “At least as good as”.

$(1, 1)$ is at least as good as $(0, 1)$

$$(1, 1) \succsim (0, 1)$$

$$(2, 1) \succsim (1, 2)$$

$$(1, 2) \succsim (2, 1)$$

Strict preference relation: when only one direction is true we write:

$$(1, 1) \succ (0, 1)$$

Indifference: when both directions are true we write:

$$(1, 2) \sim (2, 1)$$

$$(2, 1) \sim (1, 2)$$

3.1 Indifference Curves

Indifference curve is a set of bundles that are indifferent to each other.

$\sim (1, 1)$ the set of bundles indifferent to $(1, 1)$

a consumer who only cares about total scoops of ice cream:

$$\sim (1, 1)$$

$$(2, 0), (0, 2), (1, 1), (0.5, 1.5), (1.5, 0.5)$$

3.2 Chain Notation

$$p \sim p, q \sim q, r \sim r$$

$$p \succ r, p \succ q, r \sim q$$

$$p \succ r \sim q$$

Choice.

An object is best from a set if it is at least good as everything else in the set.

What is best from $\{p, q, r\}$? p

What is best from $\{q, r\}$? q, r

4 Utility

$$u(x_1, x_2) = x_1 + x_2$$

Sketch a few indifference curves for this utility function.

To get an equation for a particular indifference curve is to set the utility equal to some number:

$$x_1 + x_2 = 2$$

$$x_2 = 2 - x_1$$

4.1 MRS

Marginal Rate of Substitution. Slope of an indifference curve.

It is the amount of x_2 you are **willing** to give up to get one unit of x_1 .

This is always measured by the ratio of **marginal utilities** (partial derivatives of the utility function).

$$mu_1 = \frac{\partial(u)}{\partial x_1}, mu_2 = \frac{\partial(u)}{\partial x_2}$$

$$MRS = -\frac{mu_1}{mu_2}$$

$$x_1^2 x_2^1, x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}, \ln(x_1) + x_2, \sqrt{x_1} + x_2$$

MRS of $\sqrt{x_1} + x_2$

$$x_1^{\frac{1}{2}} + x_2$$

$$mu_1 = \frac{1}{2} x_1^{\frac{1}{2}-1} = \frac{1}{2} x_1^{-\frac{1}{2}} = \frac{1}{2} \frac{1}{x_1^{\frac{1}{2}}}$$

$$mu_2 = 1$$

$$MRS = -\frac{\frac{1}{2} \frac{1}{x_1^{\frac{1}{2}}}}{1} = -\frac{1}{2} \frac{1}{x_1^{\frac{1}{2}}}$$

What is the slope of the indifference curve at (1,1)?

$$-\frac{1}{2} \frac{1}{(1)^{\frac{1}{2}}} = -\frac{1}{2} \frac{1}{1} = -\frac{1}{2}$$

5 Solving Utility Maximization

Chapter 6: find optimal bundle given a specific set of prices and income.

$u(x_1, x_2) = \min\{x_1, \frac{1}{2}x_2\}$ and $p_1 = 1, p_2 = 1, m = 15$. What is the optimal bundle for the consumer.

Chapter 7: find marshallian demand.

$u(x_1, x_2) = \min\{x_1, \frac{1}{2}x_2\}$ find the marshallian demand.

$$x_1 = \frac{m}{p_1 + 2p_2}, x_2 = 2\frac{m}{p_1 + 2p_2}$$

5.1 Step-by-step procedure.

Identify the type of preferences:

perfect substitutes:

$$2x_1 + x_2$$

End Point + Budget

$$p_1 = 1, p_2 = 1, m = 10$$

$$-\frac{2}{1} = -\frac{1}{1}$$

perfect complements:

$$\min\{x_1, x_2\}, \min\left\{\frac{1}{2}x_1, x_2\right\}, \min\left\{2x_1, \frac{2}{3}x_2\right\}$$

No-Waste, Budget Condition

No-waste set the things inside the min equal.

$$x_1 = x_2, \frac{1}{2}x_1 = x_2, 2x_1 = \frac{2}{3}x_2$$

$$p_1x_1 + p_2x_2 = m$$

cobb-douglas:

$$x_1x_2$$

Tangency condition + Budget Constraint

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

$$p_1x_1 + p_2x_2 = m$$

quasi-linear:

$$\ln(x_1) + x_2, \sqrt{x_1} + x_2$$

Tangency condition + Budget Constraint

For $\sqrt{x_1} + x_2$

$$-\frac{1}{2} \frac{1}{x_1^{\frac{1}{2}}} = -\frac{p_1}{p_2}$$

$$p_1x_1 + p_2x_2 = m$$

$$p_1 = 1, p_2 = 1, m = 10$$

$$-\frac{1}{2} \frac{1}{x_1^{\frac{1}{2}}} = -1$$

$$\frac{1}{2} \frac{1}{x_1^{\frac{1}{2}}} = 1$$

$$\frac{1}{x_1^{\frac{1}{2}}} = 2$$

$$\frac{1}{2} = x_1^{\frac{1}{2}}$$

$$x_1 = \frac{1}{4}$$

$$x_1 + x_2 = 10$$

$$\frac{1}{4} + x_2 = 10$$

$$x_2 = \frac{39}{40}$$