1 Lagrangian

In last class we learned the conditions needed to maximize a utility function with two goods like this one:

$$u(x_1, x_2) = x_1 x_2$$

Subject to a budget constraing:

$$p_1 x_1 + p_2 x_2 = m$$

1.1 3 Goods

What if we had three goods like with this utility function?

$$u(x_1, x_2, x_3) = x_1 x_2 x_3$$

The budget constraint would be:

$$p_1 x_1 + p_2 x_2 + p_3 x_3 = m$$

How do we maximize that utility function subject to the budget constraint? The conditions are the same, the indifference curve needs to just touch but not cross into the budget set. But how do we find where those conditions are true?

The most convenient way to solve a 3+ dimensional constrained optimization problem like this is by using the Lagrange method. In this method, we convert the constrained optimization problem into an unconstrained problem by setting up a penalized version of the utility function called the Lagrangian:

Lagrangian:

$$L(x_1, x_2, x_3, \lambda) = x_1 x_2 x_3 - \lambda (p_1 x_1 + p_2 x_2 + p_3 x_3 - m)$$

Now we can maximize this by finding where it's slope is zero in every direction (including λ).

$$\frac{\partial x_1 x_2 x_3 - \lambda (p_1 x_1 + p_2 x_2 + p_3 x_3 - m)}{\partial x_1} = 0$$

$$\frac{\partial x_1 x_2 x_3 - \lambda (p_1 x_1 + p_2 x_2 + p_3 x_3 - m)}{\partial x_2} = 0$$

$$\frac{\partial x_1 x_2 x_3 - \lambda \left(p_1 x_1 + p_2 x_2 + p_3 x_3 - m\right)}{\partial x_3} = 0$$

$$\frac{\partial x_1 x_2 x_3 - \lambda \left(p_1 x_1 + p_2 x_2 + p_3 x_3 - m\right)}{\partial \lambda} = 0$$

Solving this gives us the maximium:

$$x_1 = \frac{\frac{1}{3}m}{p_1}, x_2 = \frac{\frac{1}{3}m}{p_2}, x_3 = \frac{\frac{1}{3}m}{p_3}$$

For our purposes, I want you to be familiar with this technique and know how to set up the lagrangian. But we won't focus on using this technique in this class.

2 Marshallian Demand

In the last class we maximized utility functions subject to particular budget constrains (with known price ad income)

$$u\left(x_1, x_2\right) = x_1 x_2$$

$$1x_1 + 2x_2 = 60$$

This results in an optimal bundle like this:

$$(30, 15)$$

The **marshallian demand** gives us the optimal bundle for **any** prices and income.

$$u\left(x_1, x_2\right) = x_1 x_2$$

$$p_1 x_1 + p_2 x_2 = m$$

Let's find the marshallian demand:

Tangency condition:

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

Budget condition:

$$p_1x_1 + p_2x_2 = m$$

Let's solve this. Let's simplify the tanency condition:

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

$$\frac{x_2}{x_1} = \frac{p_1}{p_2}$$

$$p_2x_2 = p_1x_1$$

$$x_2 = \frac{p_1x_1}{p_2}$$

Budget constraint:

$$p_1x_1 + p_2x_2 = m$$

$$p_1x_1 + p_2\left(\frac{p_1x_1}{p_2}\right) = m$$

$$p_1x_1 + p_1x_1 = m$$

$$2p_1x_1 = m$$

$$p_1x_1 = \frac{m}{2}$$

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

Plug this back into the tangency condition:

$$x_2 = \frac{p_1 x_1}{p_2}$$

$$x_2 = \frac{p_1 \left(\frac{\frac{1}{2}m}{p_1}\right)}{p_2} = \frac{\frac{1}{2}m}{p_2}$$

$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2}\right)$$

The consumer spends half of their money on good 1 and half on good 2.

3 Types of Goods

3.1 Income

a good is **normal** when the demand increases when income increases.

a good is **inferior** when the demand decreases when income increases.

3.2 Own Price

a good is **ordinary** when demand goes down as it's price goes up.

a good is **giffen** when demand goes up as it's price goes up.

3.3 Other Price

When the price of x_2 good goes up, and demand for x_1 goes down, we say x_1 is a **complement** for x_2.

When the price of x_2 good goes up, and demand for x_1 goes up, we say x_1 is a **substitute** for x_2.

3.4 Engel Curve

Engel curve: A plot of demand for a good (on the horizontal axis) against income (on the vertical axis.

3.5 Inverse Demand

Inverse demand: Tells us the price responsible for a consumer buying a certain amount of a good.

Suppose demand is $x_1 = \frac{10}{p_1}$. Then inverse demand is $p_1 = \frac{10}{x_1}$.

Suppose demand is $x_1 = 100 - 2p_1$. Then inverse demand is $p_1 = 50 = \frac{1}{2}x_1$.

We often also plot the inverse demand. This is a graph with price on the vertical axis and amount of the good on the horizontal axis.