Chapter 10

1 Adding Demand Curves

Aggregate Demand (Market Demand)

$x_1 = \frac{m}{p_1}$ (individual demand)

People: $J = \{a, b, c\}$

$x_{1,a}$ person a's demand for good 1.

$x_{2,b}$ person b's demand for good 2.

$x_{1,b}$ person b's demand for good 1.

Market demand for good 1:

$X_1 = x_{1,a} + x_{1,b} + x_{1,c}$

$$X_1 = \sum_{j \in J} x_{1,j}$$

$$X_2 = \sum_{j \in J} x_{2,j}$$

1.1 Example

Three people $J = \{a, b, c\}$.

$x_1 = 100 - p_1$

If the price is $p_1 = 50$. Each person demands $100 - 50 = 50$. Market demand would be 150.

$X_1 = x_{1,a} + x_{1,b} + x_{1,c}$

$$X_1 = (100 - p_1) + (100 - p_1) + (100 - p_1) = 3\,(100 - p_1)$$

$$X_1 = 300 - 3p_1$$

This asks: What is the total demand at price p_1?

Inverse Market Demand asks what price would be responsible for demand X_1?

$$X_1 = 300 - 3p_1$$

$$3p_1 = 300 - X_1$$

$$p_1 = 100 - \frac{1}{3}X_1$$

If we plug in 150 for the market demand, we should get 50.

$$p_1 = 100 - \frac{1}{3}(150) = 100 - 50 = 50$$

2 Elasticity

Notice the derivative of market demand $X_1 = 300 - 3p_1$ with respect to price is:

$$\frac{\partial (300 - 3p_1)}{\partial p_1} = -3$$

$$\frac{\partial (100 - p_1)}{\partial p_1} = -1$$

Elasticity. An elasticity measures a percentage change in some variable for a 1% change in another variable.

Change to percent change:

$$\frac{\Delta x}{\Delta p}$$

Suppose x goes from 100 to 90

Price went from 2 to 3.

$$\frac{\Delta x}{\Delta p} = \frac{90 - 100}{3 - 2} = -\frac{10}{1}$$

$$\frac{\frac{\Delta x}{x}}{\frac{\Delta p}{p}} = \frac{\frac{-10}{100}}{\frac{1}{2}} = -\frac{\frac{1}{10}}{\frac{1}{2}} = -\frac{2}{10} = -\frac{1}{5}$$

Since price went up by 50% and demand went down by 10% it is as if here that demand decreases by $\frac{1}{5}$% for every 1% increase in price.

2.1 Derivative Form

$$\frac{\frac{\Delta x}{x}}{\frac{\Delta p}{p}} = \frac{\frac{\partial x}{x}}{\frac{\partial p}{p}} = \frac{\partial x}{\partial p}\frac{p}{x} = \epsilon$$

Derivative $\frac{\partial x}{\partial p}$ mutliply it by a fraction that has the variable flipped $\frac{p}{x}$.

2.2 Price Elasticity

$$\epsilon_{1,1} = \frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1}$$

$x_1 = \frac{m}{p_1}$

$$\frac{\partial\left(x_1\right)}{\partial p_1}\frac{p_1}{x_1} = \frac{\partial\left(\frac{m}{p_1}\right)}{\partial p_1}\frac{p_1}{\frac{m}{p_1}}$$

$$\frac{m}{p_1} = m\left(p_1\right)^{-1}$$

Derivative of this is:

$$-1mp_1^{-2} = -1\frac{m}{p_1^2}$$

$$\frac{\partial\left(x_1\right)}{\partial p_1}\frac{p_1}{x_1} = \frac{\partial\left(\frac{m}{p_1}\right)}{\partial p_1}\frac{p_1}{\frac{m}{p_1}}$$

$$\frac{\partial\left(x_1\right)}{\partial p_1}\frac{p_1}{x_1} = -1\frac{m}{p_1^2}\frac{p_1}{\frac{m}{p_1}} = -1\frac{m}{p_1^2}\frac{p_1^2}{m} = -1$$

$$\epsilon_{1,1} = -1$$

For every 1% in price, demand decreases by 1%.

2.2.1 Categories

Elastic: price elasticity is < -1.

$$\epsilon_{1,1} = -10$$

When price goes up by 1% demand decreaes by 10%

Very responsive behavior in terms of price.

Normally this happens when there are a lot of substitues available for a good.

Inelastic: price elasticity is > -1

$$\epsilon_{1,1} = -0.01$$

For a 1% increase in price demand decreases by 0.01%.

Weak relatoinship between price and demand.

Necessicities, addictive goods, goods with not good subtitutes.

Unit Elastic

When elasticity is -1.

Income Elasticity Measure how demand changes in percentage terms when income goes up by 1%.

$$x_1 = \frac{m}{p_1}$$

$\eta_1 = \frac{\partial x_1}{\partial m} \frac{m}{x_1}$

$$\frac{\partial \left(\frac{m}{p_1}\right)}{\partial m} \frac{m}{\frac{m}{p_1}}$$

$$\eta_1 = \frac{1}{p_1} \frac{m}{\frac{m}{p_1}} = \frac{1}{p_1} \frac{1}{\frac{1}{p_1}} = \frac{1}{p_1} p_1 = 1$$

When income goes up by 1% demand goes up by 1%.

2.3 Cross price elasticity

$\epsilon_{1,2} = \frac{\partial x_1}{\partial p_2} \frac{p_2}{x_1}$

$$x_1 = \frac{m}{p_1 + p_2}$$

What happens to the demand for good 1 when the price of good 2 goes up by 1%?

$$\epsilon_{1,2} = \frac{\partial \left(\frac{m}{p_1+p_2} \right)}{\partial p_2} \frac{p_2}{\frac{m}{p_1+p_2}}$$

$$\frac{m}{p_1 + p_2} = m \left(p_1 + p_2\right)^{-1} = -1m \left(p_1 + p_2\right)^{-2}$$

$$= -\frac{m}{\left(p_1 + p_2\right)^2}$$

$$-\frac{m}{\left(p_1 + p_2\right)^2} \frac{p_2}{\frac{m}{p_1+p_2}} = -\frac{m}{\left(p_1 + p_2\right)^2} \frac{p_2 \left(p_1 + p_2\right)}{m}$$

$$= -\frac{p_2}{p_1 + p_2}$$