Chapter 10

1 Adding Demand Curves

Individual demand $x_1 = 10 - p_1$

People: $J = \{a, b, c\}$

$x_{1,j}$ demand for good 1, person j.

$x_{1,a}$ person a good 1

$x_{2,b}$ person b good 2

$x_{1,c}$ person b good 2

Market demand for good one:

$$X_1 = x_{1,a} + x_{1,b} + x_{1,c}$$

Suppose individual demand is $x_1 = 10 - p_1$ and there are three consumers. If $p_1 = 5$

Market demand is 15.

$$X_1 = (10 - p_1) + (10 - p_1) + (10 - p_1)$$

$$= 3(10 - p_1) = 30 - 3p_1$$

$x_{1,a} = 10 - p_1$

$x_{1,b} = 10 - 2p_1$

$x_{1,c} = 10 - 3p_1$

$$X_1 = (10 - p_1) + (10 - 2p_1) + (10 - 3p_1)$$

$$= 30 - 6p_1$$

1.1 Inverse Demand

$$X_1 = 30 - 6p_1$$

What total amount is purchased at price p_1?

$$X_1 = 30 - 6p_1$$

$$6p_1 = 30 - X_1$$

$$p_1 = 5 - \frac{1}{6}X_1$$

Inverse (market) demand

If $p_1 = 2$ then demand is $X_1 = 30 - 6\,(2) = 30 - 12 = 18$

What price is responsible for consumers buying total of 18 units?

$$p_1 = 5 - \frac{1}{6}X_1$$

$$p_1 = 5 - \frac{1}{6}\,(18)$$

$$p_1 = 5 - 3 = 2$$

2 Elasticity

An elasticity is a measurement of a relationship between variables purely in percentage terms.

Change to percent change:

Price of a good goes from 2 to 3.

Demand drops from 100 to 90.

$$\frac{\Delta x}{\Delta p} = \frac{-10}{1}$$

$$\%\Delta p = \frac{\Delta p}{p} = \frac{1}{2}$$

50% increase in price.

$$\frac{\Delta x}{x} = \frac{-10}{100} = -0.1$$

10% decrease in demand.

$$\frac{\frac{\Delta x}{x}}{\frac{\Delta p}{p}} = \frac{-0.1}{0.5} = -\frac{1}{5}$$

For every 1% increase in price demand went down by $\frac{1}{5}$%.

2.1 Derivative Form

$$\frac{\frac{\Delta x}{x}}{\frac{\Delta p}{p}}$$

To get a "percentage derivative" which we call an elasticity, turn the finite changes Δ into infitessimally smally changes ∂

$$\frac{\frac{\partial x}{x}}{\frac{\partial p}{p}} = \frac{\partial x}{\partial p}\frac{p}{x}$$

Specifically this is the "price **elasticity** of demand"

(Own) Price Elasticity

$$\varepsilon_{1,1} = \frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1}$$

Cross-Price Elasticity

$$\varepsilon_{1,2} = \frac{\partial x_1}{\partial p_2}\frac{p_2}{x_1}$$

Income Elasticity

$$\eta_1 = \frac{\partial x_1}{\partial m}\frac{m}{x_1}$$

2.2 Price Elasticity

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

$$\epsilon_{1,1} = \frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1} = \frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial p_1}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}}$$

$$\epsilon_{1,1} = \frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial p_1}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}}$$

$$\frac{\partial\left(\frac{1}{2}mp_1^{-1}\right)}{\partial p_1} = \frac{1}{2}m\left(-1\right)p_1^{-2} = -\frac{\frac{1}{2}m}{p_1^2}$$

$$\epsilon_{1,1} = -\frac{\frac{1}{2}m}{p_1^2}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}} = -\frac{\frac{1}{2}m}{p_1^2}\frac{p_1^2}{\frac{1}{2}m} = -1$$

For a 1% increase in price, demand goes down by 1%.

2.2.1 Categories

Unit Elastic Demand: price elasticity is -1

Elastic: price elasticity is < -1

Suppose it is -50. 1% increase in price leads to a 50% decrease in demand.

This might happen when there are very good substitues for goods.

Inelastic: price elasticity is > -1

-0.01 a 1% increase in price leads to a 0.01% decrease in demand.

addictive goods, necessities, medicine

2.2.2 Example $\frac{m}{p_1}$

2.3 Cross-Price Elasticity

2.3.1 Example $\frac{m}{p_1+p_2}$

$$\frac{\partial\left(\frac{m}{p_1+p_2}\right)}{\partial p_2}\frac{p_2}{\frac{m}{p_1+p_2}}$$

$$\frac{\partial\left(m\left(p_1+p_2\right)^{-1}\right)}{\partial p_2}=m\left(-1\right)\left(p_1+p_2\right)^{-2}\left(1\right)$$

$$=-\frac{m}{\left(p_1+p_2\right)^2}$$

$$-\frac{m}{\left(p_1+p_2\right)^2}\frac{p_2}{\frac{m}{p_1+p_2}}$$

$$=-\frac{1}{\left(p_1+p_2\right)^2}\frac{p_2}{\frac{1}{p_1+p_2}}$$

$$=-\frac{p_2}{\frac{\left(p_1+p_2\right)^2}{p_1+p_2}}=-\frac{p_2}{p_1+p_2}$$