1 Chapter 12 Production

f maps (x_1, x_2) into an amount of output.

Baker bakes pies using apples x_1 and crusts x_2.

2 apples and 1 crust for a pie.

$$f(2,1) = 1, f(3,1) = 1, f(4,2) = 2$$

$$f(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

Cobb Douglass

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$f(4,4) = (4)^{\frac{1}{2}} (4)^{\frac{1}{2}} = 4$$

$$f(16,1) = 16^{\frac{1}{2}} 1^{\frac{1}{2}} = 4$$

The the number that results from the production function has a maninful magnitude.

$$f(16,1) = 4$$

$$f(5,5) = 5$$

1.1 Short-Run / Long-Run Production

Short run production means that one or more of the inputs are fixed at some value.

In the short run suppose $x_1 = 4$ but you can adjust x_2.

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$f(4, x_2) = 2x_2^{\frac{1}{2}}$$

A short run production function will always less efficient that a long-run production function.

1.2 Isoquants

For consumers: $(4,4) \sim (16,1)$ two bundles are on the same indifference curve.

$$f(4,4) = (4)^{\frac{1}{2}} (4)^{\frac{1}{2}} = 4$$

$$f(16,1) = 16^{\frac{1}{2}} 1^{\frac{1}{2}} = 4$$

The input bundles $(4,4)$ and $(16,1)$ are on the same **isoquant.**

1.3 Marginal Products

For consumers, the partial derivatives of $u(x_1, x_2)$ are called the **marginal utilities**.

$$MU_1 = \frac{\partial (u(x_1, x_2))}{\partial x_1}$$

$$MRS = -\frac{MU_1}{MU_2}$$

The amount of x_2 you will give up to get one more unit of x_1.

Marginal Product of input 1. The additional amount of output you get by increasing x_1 by one unit.

$$MP_1 = \frac{\partial (f(x_1, x_2))}{\partial x_1}$$

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$MP_1 = \frac{\partial \left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} \right)}{\partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$MP_1 = \frac{1}{2} \frac{x_2^{\frac{1}{2}}}{x_1^{\frac{1}{2}}} = \frac{1}{2} \frac{\sqrt{x_2}}{\sqrt{x_1}}$$

For example at the bundle $(4,4)$

$$MP_1 = \frac{1}{2} \frac{\sqrt{4}}{\sqrt{4}} = \frac{1}{2} \frac{2}{2} = \frac{1}{2}$$

Roughly, one more unit of x_1 will increase output by $\frac{1}{2}$ unit.

1.4 Diminishing Marginal Product

As you increase one input without adjusting the other, does the extra output you get decrease?

$$f(4,4) \to f(5,4) \to f(6,4) \to f(7,4)$$

$$f(5,4) - f(4,4) = 0.47214$$

$$f(6,4) - f(5,4) = 0.42684$$

$$f(7,4) - f(6,4) = 0.39252$$

As we continue to add more x_1 we get less and less additional output.

$$MP_1 = \frac{1}{2}\frac{\sqrt{x_2}}{\sqrt{x_1}}$$

As x_1 increases, MP_1 decreases. This has **diminishing marginal product** for x_1.

1.5 Technical Rate of Substitution

MRS for consumers, which measures the slope of the indifference curve. TRS (technical rate of substitution) for producer.

$$MRS = -\frac{MU_1}{MU_2}$$

$$TRS = -\frac{MP_1}{MP_2}$$

This measures the slope of the isoquant at some point and we interpret it as (to produce the same output) **the amount you can decrease x_2 by if you increase x_1 by one unit.**

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$MP_1 = \frac{1}{2}\frac{\sqrt{x_2}}{\sqrt{x_1}}$$

$$MP_2 = \frac{1}{2}\frac{\sqrt{x_1}}{\sqrt{x_2}}$$

$$TRS = -\frac{\frac{1}{2}\frac{\sqrt{x_2}}{\sqrt{x_1}}}{\frac{1}{2}\frac{\sqrt{x_1}}{\sqrt{x_2}}} = -\frac{\frac{\sqrt{x_2}}{\sqrt{x_1}}}{\frac{\sqrt{x_1}}{\sqrt{x_2}}} = - - \frac{\sqrt{x_2}}{\sqrt{x_1}}\frac{\sqrt{x_2}}{\sqrt{x_1}} = -\frac{x_2}{x_1}$$

$(4, 4)$. The TRS is -1 we continue producing the same amount of output if we give up 1 unit of x_2 while increasing x_1 by one unit.

1.6 Returns to Scale

$$f(4, 4) \rightarrow f(5, 5) \rightarrow f(6, 6) \rightarrow f(7, 7)$$

$$f(4, 4) = 4, f(8, 8) = 8$$

Doubled both inputs and this led to double the amount of output.

Constant returns to scale:

for some $t > 1$

$$f(tx_1, tx_2) = tf(x_1, x_2)$$

$$x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$f(4, 4) = 4$$

$t = 2$

$$f(2 * 4, 2 * 4) = f(8, 8) = 8 = 2f(4, 4)$$

Linear/Constant Returns to Scale.

$$f\left(tx_1, tx_2\right) = tf\left(x_1, x_2\right)$$

Another production fucntion:

$$x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$$

$$f\left(4, 4\right) = 2.$$

$$f\left(8, 8\right) = 2.82843$$

$$f\left(1, 8\right), f\left(2, 16\right)$$

$$f\left(1, 8\right) = 1.68179$$

$$f\left(2, 16\right) = 2.37841$$

Doubling the inputs led to less than double the output.

Decreasing returns to scale.

$$f\left(tx_1, tx_2\right) < tf\left(x_1, x_2\right)$$

One more production function:

$$x_1^2 x_2^2$$

$$f\left(4, 4\right), f\left(8, 8\right)$$

$$f\left(4, 4\right) = 256$$

$$f\left(8, 8\right) = 4096$$

Doubling the inputs led to more than double the output. **Increasing returns to scale.**

$$f\left(tx_1, tx_2\right) > tf\left(x_1, x_2\right)$$