$f\left(x_1, x_2\right) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$

Firm wants to produce 4 units of output. $y = 4$.

What pair $\left(x_1, x_2\right)$ produces $y = 4$ in the cheapest way?

$w_1 = 1, w_2 = 1$

Cost of any bundle is $x_1 + x_2$.

$$(1, 16)$$

$$f\left(1, 16\right) = 1 * 4 = 4$$

The cost of this bundle is:

$$1 + 16 = 17$$

Another bundle that meets the production constraint is $4, 4$.

$$f\left(4, 4\right) = \sqrt{4} * \sqrt{4} = 4$$

Cost of this bundle is

$$4 + 4 = 8$$

We know the optimal bundle meets two conditions:

$$TRS = -\frac{w_1}{w_2}$$

$$-\frac{MP_1}{MP_2} = -\frac{w_1}{w_2}$$

$$TRS = -\frac{\frac{\partial\left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}\right)}{\partial x_1}}{\frac{\partial\left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}\right)}{\partial x_2}} = -\frac{\frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}}{\frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}} = -\frac{\frac{1}{2} x_1^{\frac{1}{2}} x_1^{\frac{1}{2}}}{\frac{1}{2} x_2^{\frac{1}{2}} x_2^{\frac{1}{2}}} = -\frac{x_1}{x_2}$$

$$TRS = -\frac{w_1}{w_2}$$

$$-\frac{x_1}{x_2} = -\frac{1}{1}$$

$$x_1 = x_2$$

This is the simplified expressions for all bundles that meet the tangency condition. Plug this into to production constraint.

$$x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = 4$$

$$x_1^{\frac{1}{2}} x_1^{\frac{1}{2}} = 4$$

$$x_1 = 4$$

$$x_2 = 4$$

$$(4, 4)$$

0.1 Another Problem

Suppose $w_1 = 1, w_2 = 2$. $f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$. Produce 4 units of output.

Two constraints are:

Production constraint:

$$x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = 4$$

Tangency condition:

$$-\frac{x_2}{x_1} = -\frac{1}{2}$$

$$x_2 = \frac{1}{2} x_1$$

Plug this into the production constraint:

$$x_1^{\frac{1}{2}}\left(\frac{1}{2}x_1\right)^{\frac{1}{2}} = 4$$

$$x_1^{\frac{1}{2}}x_1^{\frac{1}{2}}0.71 = 4$$

$$x_1 = 5.6338$$

$$x_2 = 2.8169$$

$$(5.6338, 2.8169)$$

The cost of this bundle is

$$5.6338 + 2\,(2.8169) \quad = \quad 11.2676$$

It produces:

$$3.9837$$

Is this cheaper than $(4, 4)$

$$4 + 2 * 4 = 12$$

0.2 Perfect Complements Production function.

$$f\,(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

Prices are both $w_1 = 1, w_2 = 1$. What is the cheapest way of producting output 4?

Production constraint:

$$min\left\{\frac{1}{2}x_1, x_2\right\} = 4$$

Cheapest bundle must respect the no waste condition:

$$\frac{1}{2}x_1 = x_2$$

Plug no waste into the production constraint:

$$min\left\{\frac{1}{2}x_1, \frac{1}{2}x_1\right\} = 4$$

$$\frac{1}{2}x_1 = 4$$

$$x_1 = 8$$

$$x_2 = 4$$

$$(8, 4)$$

1 Condition factor demands and cost functions.

What is the cheapest way of producting output y if prices are w_1 and w_2?

Production constraint:

$$min\left\{\frac{1}{2}x_1, x_2\right\} = y$$

No waste condition:

$$\frac{1}{2}x_1 = x_2$$

Plug no waste into the production constraint:

$$min\left\{\frac{1}{2}x_1, \frac{1}{2}x_1\right\} = y$$

$$\frac{1}{2}x_1 = y$$

$$x_1 = 2y$$

$$x_2 = y$$

$$(2y, y)$$

The cost of the cheapest way of producing output is called the **cost function**.

$$c(y) = w_1(2y) + w_2 y = 2w_1 y + w_2 y$$

$$c(y) = (2w_1 + w_2)\, y$$

2 Profit in terms of y.

$$py - c(y)$$

$(16.67)\,(350)\,0.5$

$$(16.67)\,(350)\,0.5 = 2917.25$$

$$(35)\,(350)\,0.5 = 6125.$$

$$(20 * 350) = 7000$$

$$2917.25 + 6125 + 7000 = 16042.3$$

$$12500 + 4166.67 = 16666.7$$

$$16666.7 - 16042.3 = 624.4$$