Exercise 6

$$u(w,h) = wh$$

$$2w + 2h = 20$$

$$\left(\frac{\frac{1}{2}20}{2}, \frac{\frac{1}{2}20}{2}\right)$$

$$(5,5)$$

Exercise 7

$$x_1^2 + x_2^2$$

$$p_1 = 1, p_2 = 2, m = 60$$

$$MRS = -\frac{2x_1}{2x_2} = -\frac{x_1}{x_2}$$

$$-\frac{x_1}{x_2} = -\frac{1}{2}$$

Tangency Condition:

$$2x_1 = x_2$$

Budget condition:

$$1x_1 + 2x_2 = m$$

$$1x_1 + 2(2x_1) = 60$$

$$5x_1 = 60$$

$$x_1 = \frac{60}{5} = 12$$

$$24 = \frac{2}{5}60 = x_2$$

$$(12, 24)$$

$$u(12, 24) = 12^2 + 24^2 \quad = \quad 720$$

$$u(60, 0) = 60^2 + 0^2 = 3600$$

$$u(0, 30) = 0 + 30^2 = 900$$

Exercise 9

$$u(x_1, x_2, x_3) = ln(x_1) + ln(x_2) + ln(x_3)$$

$$x_1 + x_2 + 2x_3 = 60$$

$$ln(x_1) + ln(x_2) + ln(x_3) - \lambda(x_1 + x_2 + 2x_3 - 60)$$

$$\frac{\partial(ln(x_1) + ln(x_2) + ln(x_3) - \lambda(x_1 + x_2 + 2x_3 - 60))}{\partial x_1} = 0$$

$$\frac{\partial(ln(x_1) + ln(x_2) + ln(x_3) - \lambda(x_1 + x_2 + 2x_3 - 60))}{\partial x_2} = 0$$

$$\frac{\partial(ln(x_1) + ln(x_2) + ln(x_3) - \lambda(x_1 + x_2 + 2x_3 - 60))}{\partial x_3} = 0$$

$$\frac{\partial \left(ln\left(x_1 \right) + ln\left(x_2 \right) + ln\left(x_3 \right) - \lambda \left(x_1 + x_2 + 2x_3 - 60 \right) \right)}{\partial \lambda} = 0$$

Exercise 14.

$$MRS = -\frac{\frac{\partial \left(x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} \right)}{\partial x_1}}{\frac{\partial \left(x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} \right)}{\partial x_2}} = -\frac{x_2}{2x_1} = -\frac{1}{2}\frac{x_2}{x_1}$$

$$-\frac{x_2}{2x_1} = -\frac{1}{1}$$

$$x_2 = 2x_1$$

$$x_1 + x_2 = 60$$

$$(20, 40)$$

Exercise 8

$$\frac{1}{2}x_1 = x_2$$

$$p_1 = 1, p_2 = 2, m = 60$$

10 apples free if you buy two crusts, what do you buy.

Don't get the promotion $x_2 < 2$.

$$x_1 + 2x_2 = 60$$

Do get the promotion $x_2 \geq 2$

If I end up with $x_1 = 11$ then I only had to buy $1.(11 - 10) = 1$

$$(x_1 - 10) + 2x_2 = 60$$

$$\frac{1}{2}x_1 = x_2$$

$$(x_1 - 10) + 2\left(\frac{1}{2}x_1\right) = 60$$

$$x_1 - 10 + x_1 = 60$$

$$x_1 = 35$$

$$x_2 = \frac{45}{2} = \frac{35}{2} = 17.5$$

$$(35, 17.5)$$

Exercise 8 Chapter 7

$$ln\left(x_1\right) + x_2$$

$p_1x_1 + p_2x_2 = m$

$$MU_1 = \frac{\partial\left(ln\left(x_1\right) + x_2\right)}{\partial x_1} = \frac{1}{x_1}$$

$$MU_2 = \frac{\partial\left(ln\left(x_1\right) + x_2\right)}{\partial x_2} = 1$$

$$MRS = -\frac{\frac{1}{x_1}}{1} = -\frac{1}{x_1}$$

$$-\frac{1}{x_1} = -\frac{p_1}{p_2}$$

$$x_1 = \frac{p_2}{p_1}$$

Plug this into the budget constraint:

$$p_1\left(\frac{p_2}{p_1}\right) + p_2 x_2 = m$$

$$p_2 + p_2 x_2 = m$$

$$x_2 = \frac{m - p_2}{p_2}$$

$$\left(\frac{p_2}{p_1}, \frac{m - p_2}{p_2}\right)$$