1 Profit Maximization

Find the cheapest way of producing **any** amount of output y.

$$c(y) = y^2$$

1.1 Profit Function

Write the profit function purely in terms of output:

$$\pi(y) = p(y)\,y - c(y)$$

The simplest assumption is **price taking**. The firm assumes they will get p per unit of output regardless of how much they produce.

$$\pi(y) = py - c(y)$$

This is most reasonable when each firm is a very small amount of a very large market. ("Perfect Competition").

1.2 Maximizing

$$c(y) = y^2$$

The marginal cost:

$$mc(y) = \frac{\partial\left(y^2\right)}{\partial y} = 2y$$

Let suppose $p = 10$. Under price-taking, the profit function:

$$\pi(y) = 10y - y^2$$

Here is profit for y from 1 to 8:

$$\{9, 16, 21, 24, 25, 24, 21, 16\}$$

$$\frac{\partial\left(10y - y^2\right)}{\partial y} = 10 - 2y$$

$$10 - 2y = 0$$

$$2y = 10$$

$$y^* = 5$$

Plug that into the profit function to find the maximum profit:

$$\pi(5) = 10 * 5 - 5^2 = 50 - 25 = 25$$

1.3 What can go wrong?

$$c(y) = 2y$$

$$mc(y) = 2$$

$$p = 10$$

$$\pi(y) = 10y - 2y = 8y$$

Suppose cost is:

$$c(y) = \sqrt{y}$$

The marginal cost:

$$mc(y) = \frac{\partial\left(y^{\frac{1}{2}}\right)}{\partial y} = \frac{1}{2}y^{-\frac{1}{2}} = \frac{1}{2\sqrt{y}}$$

This is decreasing marginal cost.

Suppose

$$p = 10$$

$$\pi(y) = 10y - \sqrt{y}$$

$$\frac{\partial\left(10y - \sqrt{y}\right)}{\partial y} = 10 - \frac{1}{2\sqrt{y}}$$

$$10 - \frac{1}{2\sqrt{y}} = 0$$

$$10 = \frac{1}{2\sqrt{y}}$$

$$\sqrt{y} = \frac{1}{20}$$

$$y = \frac{1}{400}$$

This appears to be optimal.

$$\pi\left(\frac{1}{400}\right) = 10\left(\frac{1}{400}\right) - \sqrt{\frac{1}{400}} = -0.025$$

$$\{0., 9., 18.5858, 28.2679, 38., 47.7639\}$$