1 Monopolies

Price-taking assumption. Price is not a function of output.

$$\pi\left(q\right) = pq - c\left(q\right)$$

For a monopoly, their individual quantity **is the market quantity.**

$$\pi\left(q\right) = p\left(q\right)q - c\left(q\right)$$

$p\left(y\right)$ (the way price depends on output) is the inverse demand.

1.1 Example

Suppose monopoly has a cost function of $c\left(q\right) = 10q$.

Market demand function is:

$$q = 100 - p$$

Inverse demand (solve for p).

$$p = 100 - q$$

a function that determines what price consumers would be willing to pay for quantity q. This is our $p\left(q\right)$

The profit function for this firm in terms output q is given by:

$$\pi\left(q\right) = \left(100 - q\right)q - 10q$$

Let's simplify this:

$$\pi\left(q\right) = 100q - q^2 - 10q$$

$$\pi\left(q\right) = 90q - q^2$$

$$\{0., 800., 1400., 1800., 2000., 2000., 1800., 1400., 800., 0., -1000.\}$$

This function has the concave shape that ensures that when we look for where the slope is zero, we find a maximum instead of a minimum.

To find the profit maximizing quatity, look for where the marginal profit is zero:

$$\frac{\partial\left(90q-q^{2}\right)}{\partial q}=0$$

$$90-2q=0$$

$$q^{*}=45$$

The profit they earn for producing 45 is:

$$90\left(45\right)-\left(45\right)^{2} \quad = \quad 2025$$

What price do they charge? Plug the quantitity into the inverse demand:

$$p^{*}=\left(100-45\right)=55$$

They sell 45 at price of 55 and earn 2025.