1 Price Discrimination

1.1 Types

First-Degree: You charge everyone their full willingness to pay.

Uber, Algorithmic Pricing

Second-Degree: Create different packages, levels of service, quality, etc at different prices. And let the consumers decide what they will buy.

First-Class / Coach.

Whiskey

Car Trims

Fast-Pass Amusement Park

Fancy Credit Cards

Third-Degree: Different Groups you can identify and charge them different amount.

Student Tickets

Vet. Discounts

Resident Discounts

1.2 Example Third Degree

Non-students: $q_n = 100 - p_n$

Students: $q_s = 100 - 2p_s$

$c(q) = 0$

Total Demand if we change p to everyone:

$$(100 - p) + (100 - 2p) = 200 - 3p$$

$$q = 200 - 3p$$

$$3p = 200 - q$$

$$p = \frac{200}{3} - \frac{1}{3}q$$

Profit function treating both groups as one:

$$\pi(q) = \left(\frac{200}{3} - \frac{1}{3}q\right) q$$

$$= \frac{200}{3}q - \frac{1}{3}q^2$$

Where is the slope zero? This will give the optimal q to sell to maximize profit:

$$\frac{\partial\left(\frac{200}{3}q - \frac{1}{3}q^2\right)}{\partial q} = 0$$

$$\frac{200}{3} - \frac{2}{3}q = 0$$

$$\frac{200}{3} = \frac{2}{3}q$$

$$100 = q^*$$

Since the inverse demand is $p = \frac{200}{3} - \frac{1}{3}q$ you can charge:

$$\frac{200}{3} - \frac{1}{3}(100) = \frac{100}{3} \approx \$33.33$$

Total profit is quantity times price (zero costs)

$$100 * \frac{100}{3} = \frac{10000}{3} \approx 3333.33$$

1.3 Treat Them as Independent Markets

Non-students: $q_n = 100 - p_n$

Non-student Inverse Demand: $p_n = 100 - q_n$

$$\pi_n(q_n) = (100 - q_n) q_n$$

Maximizing this by finding where the slope is zero gives us:

$$q_n^* = 50$$

$$p_n^* = 50$$

$$\pi_n^* = 50 * 50 = 2500$$

Students: $q_s = 100 - 2p_s$

Student Inverse Demand: $p_s = 50 - \frac{1}{2}q_s$

$$\pi_s\left(q_s\right) = \left(50 - \frac{1}{2}q_s\right)q_s$$

$$q_s^* = 50$$

$$p_s^* = 25$$

$$\pi_s^* = 50 * 25 = 1250$$

Total profit for both grous:

$$\pi^* = \pi_n^* + \pi_s^* = 2500 + 1250 = 3750$$