1 Cournot Model

$$\pi(q) = p(q)\,q - c(q)$$

Price taking: $p(q) = p$

Monopoly: $p(q)$ is the **inverse demand**: how much will consumers pay to buy q?

1.1 Monopoly

$q = 100 - p$, $c(q) = 10q$

$$q = 100 - p$$

Solve for the inverse demand:

$$p = 100 - q$$

$$\pi(q) = (100 - q)\,q - 10q$$

$$= 100q - q^2 - 10q = 90q - q^2$$

$$\frac{\partial\left(90q - q^2\right)}{\partial q} = 0$$

$$90 - 2q = 0$$

$$q^* = 45$$

Plug this into the inverse demand to get price:

$$p^* = 100 - 45 = 55$$

$$\pi(45) = (55)\,45 - 10\,(45) = 2025$$

1.2 Cournot

When we have more than one firm, the market quantity Q is the sum of the individual firms quantities.

Let's suppose we have two firms

The market quantity is the sum of the individual quantities.

$$Q = q_1 + q_2$$

Suppose the firms have chosen to produce $q_1 = 40, q_2 = 40$. The market quantity is $Q = 40 + 40 = 80$.

How much will consumers pay for 80 units?

If the inverse demand $p = 100 - Q$, they can charge $p = 20$.

Suppose we have two firms with inverse demand $p = 100 - Q$ and costs are both $c\left(q_i\right) = 10q_i$.

The profit function for firm 1:

$$\pi_1\left(q_1, q_2\right) = \left(100 - \left(q_1 + q_2\right)\right) q_1 - 10q_1$$

Compare to monopoly:

$$\pi\left(q\right) = \left(100 - q\right) q - 10q$$

1.3 One More Example:

$Q = 50 - 2p$. Each firm has cost $c\left(q_i\right) = 5q_i$.

The inverse demand:

$$2p + Q = 50$$

$$2p = 50 - Q$$

$$p = 25 - \frac{1}{2}Q$$

If this was a monopolist (only one firm).

$$\pi\left(q\right) = \left(25 - \frac{1}{2}q\right) q - 10q$$

Now suppose we have two firms:

$$\pi_1(q_1) = \left(25 - \frac{1}{2}(q_1 + q_2)\right) q_1 - 10q_1$$

1.4 Back to the other example:

$$\pi_1(q_1, q_2) = (100 - (q_1 + q_2))\, q_1 - 10q_1$$

$$= (100 - q_1 - q_2)\, q_1 - 10q_1$$

Let's simplify this:

$$= 100q_1 - q_1^2 - q_1 q_2 - 10q_1$$

$$= 90q_1 - q_1^2 - q_1 q_2$$

Suppose firm 1 thinks firm 2 will choose $q_2 = 20$.

$$= 90q_1 - q_1^2 - q_1(20)$$

$$= 70q_1 - q_1^2$$

Where does this have a slope of zero?

$$70 = 2q_1$$

$$q_1^* = 35$$

Suppose instead they were producing $q_2 = 50$

$$= 90q_1 - q_1^2 - q_1(50)$$

$$q_1^* = 20$$

For any quanitty firm 2 produces what should firm 1 choose? This is called the "best response function".

$$\frac{\partial\left(90q_1 - q_1^2 - q_1 * q_2\right)}{\partial q_1} = 0$$

$$90 - 2q_1 - q_2 = 0$$

$$2q_1 = 90 - q_2$$

$$q_1^* = \frac{90 - q_2}{2}$$

$$q_2^* = \frac{90 - q_1}{2}$$

$$q_1 = 35, q_2 = 20$$

Firm 1 has no incentive to change their behavior.

What about firm 2?

$$q_2 = \frac{90 - 35}{2} = 27.5$$

Imagine we have a pair of quantities that were both best-responses to eachother.

$$q_1 = 30, q_2 = 30$$

$$q_1^* = \frac{90 - q_2}{2}$$

$$q_2^* = \frac{90 - q_1}{2}$$

Both firms quantitis are **simeltaneously** best-responses to each other.

Nash Equilibrium: set of strategies (quantities) that are simultaneously best-responses.

The only equlibrium here is:

$$q_1 = 30, q_2 = 30$$

To solve this easily, take any best response function. Impose symmetry on the best response function to look for a quantity that is a best-response to itself:

$$q = \frac{90 - q}{2}$$

$$2q = 90 - q$$

$$3q = 90$$

$$q^* = 30$$

What price can they charge?

$$p = 100 - Q$$

$Q = 30 + 30 = 60$

$$p = 100 - 60 = 40$$

2 Back to the other example

inverse demand $p = 25 - \frac{1}{2}Q$ and costs $c(q_i) = 5q_i$

$$\pi_1(q_1, q_2) = \left(25 - \frac{1}{2}(q_1 + q_2)\right) q_1 - 5q_1$$

$$\pi_2(q_1, q_2) = \left(25 - \frac{1}{2}(q_1 + q_2)\right) q_2 - 5q_2$$

Where does either firm maximize profit. Let's do it for firm 1.

$$\left(25 - \frac{1}{2}q_1 - \frac{1}{2}q_2\right) q_1 - 5q_1$$

$$25q_1 - \frac{1}{2}q_1^2 - \frac{1}{2}q_1q_2 - 5q_1$$

$$= 20q_1 - \frac{1}{2}q_1^2 - \frac{1}{2}q_1q_2$$

$$\frac{\partial\left(20q_1 - \frac{1}{2}q_1^2 - \frac{1}{2}q_1q_2\right)}{\partial q_1} = 0$$

$$20 - q_1 - \frac{1}{2}q_2 = 0$$

$$q_1 = 20 - \frac{1}{2}q_2$$

What is the best response to 20?

$$q_1 = 20 - \frac{1}{2}20 = 20 - 10 = 10$$

To find the nash equilibrium impose symmetry on the best response function:

$$q = 20 - \frac{1}{2}q$$

$$\frac{3}{2}q = 20$$

$$q^* = \frac{40}{3}$$