

1 Problem 1 chapter 17

$$p = 50 - Q, c(q_1) = 5q_1, c(q_2) = 5q_2$$

a)

$$\pi_1(q_1, q_2) = (50 - (q_1 + q_2)) q_1 - 5q_1$$

$$\pi_2(q_1, q_2) = (50 - (q_1 + q_2)) q_2 - 5q_2$$

b)

$$\pi_1(q_1, q_2) = (50 - (q_1 + q_2)) q_1 - 5q_1$$

This is maximized where the slope in terms of q_1 is zero.

$$= (50 - (q_1 + q_2)) q_1 - 5q_1$$

$$= (50 - q_1 - q_2) q_1 - 5q_1$$

$$= 50q_1 - q_1^2 - q_2q_1 - 5q_1$$

$$= 45q_1 - q_1^2 - q_2q_1$$

$$\frac{\partial (45q_1 - q_1^2 - q_2q_1)}{\partial q_1} = 45 - 2q_1 - q_2$$

Set this to zero and solve for q_1 to the the optimal q_1 .

$$45 - 2q_1 - q_2 = 0$$

$$45 - q_2 = 2q_1$$

$$q_1 = \frac{45 - q_2}{2}$$

$$\pi_2 = (50 - (q_1 + q_2)) q_2 - 5q_2$$

$$q_2 = \frac{45 - q_1}{2}$$

This is the best-response function for firm 1.

c)

$$q_1 = \frac{45 - (25)}{2} = \frac{20}{2} = 10$$

If firm 2 chooses 25, firm 1 should choose 10.

d)

Impose symmetry on one of the response functions to look for a quantity that is a best response to itself.

$$q = \frac{45 - q}{2}$$

$$2q = 45 - q$$

$$3q = 45$$

$$q^* = 15$$

e)

$$p = 50 - (15 + 15) = 50 - 30 = 20$$

$$p^* = 20$$

f)

$$20 * 15 - 5 (15) = 225$$

$$\pi_1 (15, 15) = (50 - (15 + 15)) 15 - 5 (15) = 225$$

2 Question 2

Firm 1 chooses their quantity q_1 . Firm 2 observes q_1 and chooses q_2 .

Sequential game.

a)

$$\pi_1(q_1, q_2) = (50 - (q_1 + q_2))q_1 - 5q_1$$

$$\pi_2(q_1, q_2) = (50 - (q_1 + q_2))q_2 - 5q_2$$

b)

$$q_2 = \frac{45 - q_1}{2}$$

c)

Plug the best response of firm 2 in for q_2 since firm 1 knows this is how firm 2 responds:

$$\pi_1(q_1, q_2) = \left(50 - \left(q_1 + \frac{45 - q_1}{2}\right)\right)q_1 - 5q_1$$

$$= \left(50 - \left(q_1 + \frac{45 - q_1}{2}\right)\right)q_1 - 5q_1$$

$$= \left(50 - q_1 - \frac{45 - q_1}{2}\right)q_1 - 5q_1$$

$$= \left(50 - q_1 - \left(\frac{45}{2} - \frac{q_1}{2}\right)\right)q_1 - 5q_1$$

$$= \left(50 - q_1 - \frac{45}{2} + \frac{q_1}{2}\right)q_1 - 5q_1$$

$$= 50q_1 - q_1^2 - \frac{45}{2}q_1 + \frac{1}{2}q_1^2 - 5q_1$$

$$= \frac{45}{2}q_1 - \frac{1}{2}q_1^2$$

To maximize this profit function find where the slope is zero:

$$\frac{\partial \left(\frac{45}{2}q_1 - \frac{1}{2}q_1^2 \right)}{\partial q_1} = 0$$

$$\frac{45}{2} - q_1 = 0$$

$$q_1 = 22.5$$

Find firm 2's response:

$$q_2 = \frac{45 - 22.5}{2} = 11.25$$

f) What are their profits:

$$\pi_1(22.5, 11.25) = (50 - (22.5 + 11.25))22.5 - 5(22.5)$$

$$= 253.125$$

$$\pi_2(22.5, 11.25) = (50 - (22.5 + 11.25))11.25 - 5(11.25) = 126.563$$

$$= 126.563$$

$$\left\{ \left\{ q_2 \rightarrow \frac{1}{2}(120 - q_1) \right\} \right\}$$

$$\{ \{ q_1 \rightarrow 60 \} \}$$

$$q_2 = 30$$