1 Exchange

1.1 Pies

Farmer has 60 apples

Baker has 30 crusts

Both only eat pies that require 1 crust and 2 apples.

Farmer suggests 30 apples in exchange for 15 crusts.

Both end with the bundle:

$$(15, 30)$$

What prices facilitate this trade?

2 apples for every crust. In this exchange it is as if the price of crusts is double the price of apples:

$$p_1 = 2, p_2 = 1$$

An exchange is defined by the relative prices.

$$p_1 = 4, p_2 = 2$$

What we look for in this change is a set of prices that will facilitate an agreeable exchange.

1.2 Not and Equilibrium

Same endowments. 60 apples for the farmer. and 30 crusts for the baker.

Baker eats pies $min\left\{x_1, \frac{1}{2}x_2\right\}$

Farmer eats anything $x_1 + x_2$

Baker suggests “For every crust I give you, you give me two apples”.

$p_1 = 2, p_2 = 1$

Baker has 30 crusts. Worth 60. The price of a pie is 4. What bundle do they demand at these prices?

$$(15, 30)$$

$$(0, 60)$$

The total demand for crusts is 15 total demand for apples is 90. Because the demands don't sum to the endowment, there is no agreeable exchange at these prices.

Would the farmer agree to any exchange here?

The farmer would not agree to any trade at these prices.

$p_1 = 2, p_2 = 1$ is not a set of equilibrium prices.

Let's try $p_1 = 1, p_2 = 1$. What exchange happens?

We know that the baker still needs to me the no-waste condition:

Suppose the baker suggests giving up 15 crusts in exchange for 15 apples.

$$(10, 20)$$

$$(20, 40)$$

This is an equilibrium. The prices are $p_1 = 1, p_2 = 1$ and the equilibrim allocation is $(10, 20)$ $(20, 40)$.

1.3 Formal Example

Person a's demand for good 1: $x_{1,a}$

Person a's demand for good 2: $x_{2,a}$

Person b's demand for good 1: $x_{1,b}$

Person b's demand for good 2: $x_{2,b}$

Person a's endowment for good 1: $\omega_{1,a}$

Person a's endowment for good 2: $\omega_{2,a}$

Person b's endowment for good 1: $\omega_{1,b}$

Person b's endowment for good 2: $\omega_{2,b}$

$u\left(x_{1,a}, x_{2,a}\right) = x_{1,a}x_{2,a}$

$u\left(x_{1,a}, x_{2,a}\right) = x_{1,b}x_{2,b}$

$\omega_{1,a} = 30, \omega_{2,a} = 0$

$\omega_{1,b} = 0, \omega_{2,b} = 60$

Under a particular set of prices, what does each consumer demand?

What is the demand for person a under prices p_1, p_2?

Person a has the budget equation:

$$p_1 x_{1,a} + p_2 x_{2,a} = p_1 \omega_{1,a} + p_2 \omega_{2,a}$$

Person b has the budget equation:

$$p_1 x_{1,b} + p_2 x_{2,b} = p_1 \omega_{1,b} + p_2 \omega_{2,b}$$

Consumer $a's$ utility is $u\left(x_{1,a}, x_{2,a}\right) = x_{1,a} x_{2,a}$

What is their marshallian demand for person a for goods 1 and 2?

$$\left(\frac{\frac{1}{2}\left(30 p_1\right)}{p_1}, \frac{\frac{1}{2}\left(30 p_1\right)}{p_2}\right)$$

The marshallian demand for good

$$\left(\frac{\frac{1}{2}\left(60 p_2\right)}{p_1}, \frac{\frac{1}{2}\left(60 p_2\right)}{p_2}\right)$$

What prices create an equilibrium in this market?

The market clearing condition for good 1 :

$$\frac{\frac{1}{2}\left(30 p_1\right)}{p_1} + \frac{\frac{1}{2}\left(60 p_2\right)}{p_1} = 30$$

For good 2:

$$\frac{\frac{1}{2}\left(30 p_1\right)}{p_2} + \frac{\frac{1}{2}\left(60 p_2\right)}{p_2} = 60$$

To solve for an equilibrium, it is convenient to normalize on of the prices to 1 and solve for the other price.

Let's find the p_2 that clears the market for good 1 when $p_1 = 1$.

$$\frac{\frac{1}{2}(30)}{1} + \frac{\frac{1}{2}\left(60 p_2\right)}{1} = 30$$

$$15 + 30 p_2 = 30$$

$$30p_2 = 15$$

$$p_2 = \frac{1}{2}$$

If the price of $p_1 = 1$ then to find an agreeable exchange in this economy the price of p_2 needs to be $\frac{1}{2}$.

That is, for every unit of good 1 a consumer gives up, they expect to get 2 units of good 2.

$$\left(\frac{\frac{1}{2}(30)}{1}, \frac{\frac{1}{2}(30)}{\frac{1}{2}}\right) = (15, 30)$$

$$\left(\frac{\frac{1}{2}\left(60\frac{1}{2}\right)}{1}, \frac{\frac{1}{2}\left(60\frac{1}{2}\right)}{\frac{1}{2}}\right) = (15, 30)$$

Under these prices, both consumers demand $(15, 30)$.

What exhange did the consumers make.

Person a started with 30 of good 1.

Person b started with 60 of good 2.

$$(15, 30)$$

$$(15, 30)$$

The equilibrium prices are $p_1 = 1, p_2 = \frac{1}{2}$ and the equilibrium allocations are for both:

$$(15, 30)$$

2 Problem 1 From Exercises

Consumer a and consumer b

$\omega_{1,a} = 20$

$\omega_{2,b} = 20$

a) What are budget equations at prices p_1, p_2

$$p_1 x_{1,a} + p_2 x_{2,a} = 20p_1$$

$$p_1 x_{1,b} + p_2 x_{2,b} = 20p_2$$

b) What are the market clering conditions?

$$x_{1,a} + x_{1,b} = 20$$

$$x_{2,a} + x_{2,b} = 20$$

c) $u\left(x_{1,a}, x_{2,a}\right) = x_{1,a} x_{2,a}$ remember that a consumer with this utility spends half of their income on each good.

$$-\frac{x_{2,a}}{x_{1,a}} = -\frac{p_1}{p_2}$$

$$p_1 x_{1,a} + p_2 x_{2,a} = 20p_1$$

$$\left(\frac{\frac{1}{2}\left(20p_1\right)}{p_1}, \frac{\frac{1}{2}\left(20p_1\right)}{p_2}\right)$$

d) $u\left(x_{1,b}, x_{2,b}\right) = min\left\{x_{1,b}, x_{2,b}\right\}$

$$x_{1,b} = x_{2,b}$$

$$p_1 x_{1,b} + p_2 x_{2,b} = 20p_2$$

Plug the first into the second:

$$p_1 x_{1,b} + p_2 x_{1,b} = 20p_2$$

$$x_{1,b}\left(p_1 + p_2\right) = 20p_2$$

$$x_{1,b} = \frac{20p_2}{p_1 + p_2}, x_{2,b} = \frac{20p_2}{p_1 + p_2}$$

$$\left(\frac{20p_2}{p_1 + p_2}, \frac{20p_2}{p_1 + p_2}\right)$$

e) Is $p_1 = 1, p_2 = 2$ and equilibrium?

To check this, see if the markets clear at these prices.

Demand for good 1 :

$$\frac{\frac{1}{2}(20p_1)}{p_1} + \frac{20p_2}{p_1 + p_2}$$

Plug these prices in:

$$\frac{\frac{1}{2}(20)}{1} + \frac{20(2)}{1+2} = 10 + \frac{40}{3} = 23.3333 \neq 20$$

There is an over-demand for good 1 and thus there will be an over-supply of good 2.

g) Assume $p_1 = 1$ what is the the equilibirum price of p_2?

$$\frac{\frac{1}{2}(20)}{1} + \frac{20(p_2)}{1+p_2} = 20$$

$$10 + \frac{20p_2}{1+p_2} = 20$$

$$\frac{20p_2}{1+p_2} = 10$$

$$20p_2 = 10 + 10p_2$$

$$10p_2 = 10$$

$$p_2 = 1$$

If $p_1 = 1$ then $p_2 = 1$.

$$(10, 10)$$

$$(10, 10)$$

3 An aside

$$x_{1,a}^2 x_{2,a}$$

$$min\left\{x_1, x_2\right\}$$

$$\left(\frac{\frac{2}{3}\left(20p_1\right)}{p_1}, \frac{\frac{2}{3}\left(20p_1\right)}{p_2}\right)$$

$$\left(\frac{20p_2}{p_1 + p_2}, \frac{20p_2}{p_1 + p_2}\right)$$

Market clearing for good 1 :

$$\frac{\frac{2}{3}\left(20p_1\right)}{p_1} + \frac{20p_2}{p_1 + p_2} = 20$$

Normalize $p_1 = 1$

$$\frac{\frac{2}{3}\left(20\right)}{1} + \frac{20p_2}{1 + p_2} = 20$$

$$p_2 = \frac{1}{2}$$