1 Public Goods

100 people share a park

Every individual has $m_i = 1000000$

Every individuals contribution is g_i

Individuals consumption is $c_i = m_i - g_i$

$u(c_i, G) = c_i + 100\sqrt{G}$

If G_{-i} is the total contributions of everone who is not i:

$G = g_i + G_{-i}$

$$= m_i - g_i + 100\sqrt{g_i + G_{-i}}$$

What contribution maximizes i's utility?

$$\frac{50}{\sqrt{g_i + G_{-i}}} - 1 = 0$$

$$50 = \sqrt{g_i + G_{-i}}$$

$$2500 = g_i + G_{-i}$$

$$g_i = 2500 - G_{-i}$$

Let's find a symmetric equilirbium where everyone contributes some amount g:

$$g = 2500 - (99g)$$

Solve this for g:

$$100g = 2500$$

$$g = 25$$

Total contributions are $G = 2500$

$$u = 1000000 - 25 + 100\sqrt{2500} = 1004975$$

1.1 Social Optimum Tax

What tax t maximizes the utility of every individual.

$$= 1000000 - t + 100\sqrt{100t}$$

$$\frac{\partial\left(1000000 - t + 100\sqrt{100t}\right)}{\partial t} = 0$$

$$\frac{500}{\sqrt{t}} - 1 = 0$$

$$500 = \sqrt{t}$$

$$t = 250000$$

$$1000000 - 250000 + 100\sqrt{100 * 250000} = 1250000$$

2 Exchange Economy

2.1 Exercise 1

Two consumers

$$x_{1,a} = \frac{\frac{1}{2}\left(p_1\omega_{1,a} + p_2\omega_{2,a}\right)}{p_1}, x_{2,a} = \frac{\frac{1}{2}\left(p_1\omega_{1,a} + p_2\omega_{2,a}\right)}{p_2}$$

$$x_{1,b} = \frac{p_1\omega_{1,b} + p_2\omega_{2,b}}{p_1 + p_2}, x_{2,b} = \frac{p_1\omega_{1,b} + p_2\omega_{2,b}}{p_1 + p_2}$$

$\omega_{1,a} = 20, \omega_{2,a} = 0, \omega_{1,b} = 0, \omega_{2,b} = 20$

$$x_{1,a} = \frac{\frac{1}{2}\left(20p_1\right)}{p_1}, x_{2,a} = \frac{\frac{1}{2}\left(20p_1\right)}{p_2}$$

$$x_{1,b} = \frac{20p_2}{p_1 + p_2}, x_{2,b} = \frac{20p_2}{p_1 + p_2}$$

a)

Market clearing conditions:

$$x_{1,a} + x_{1,b} = \omega_{1,a} + \omega_{1,b}$$

$$x_{2,a} + x_{2,b} = \omega_{2,a} + \omega_{2,b}$$

Plug in the demands and the endowments:

$$x_{1,a} = \frac{\frac{1}{2}(20p_1)}{p_1}, x_{2,a} = \frac{\frac{1}{2}(20p_1)}{p_2}$$

$$x_{1,b} = \frac{20p_2}{p_1 + p_2}, x_{2,b} = \frac{20p_2}{p_1 + p_2}$$

$$\frac{\frac{1}{2}(20p_1)}{p_1} + \frac{20p_2}{p_1 + p_2} = 20$$

$$\frac{\frac{1}{2}(20p_1)}{p_2} + \frac{20p_2}{p_1 + p_2} = 20$$

b) Is $p_1 = 1$ and $p_2 = 2$ an equilibrium?

$$\frac{\frac{1}{2}(20(1))}{(1)} + \frac{20(2)}{1+2} = 20$$

$$\frac{\frac{1}{2}(20(1))}{2} + \frac{20(2)}{1+2} = 20$$

$$23.3333 = 20$$

$$18.3333 = 20$$

Not an equilibrium, there is an over-demand of good 1 and over-supply of good 2.

c) If $p_1 = 1$ what must p_2 be in equilibrium

$$\frac{\frac{1}{2}(20)}{1} + \frac{20p_2}{1+p_2} = 20$$

$$\frac{\frac{1}{2}(20)}{p_2} + \frac{20p_2}{1+p_2} = 20$$

What p_2 clears the market for good 1?

$$10 + \frac{20p_2}{1+p_2} = 20$$

$$\frac{20p_2}{1+p_2} = 10$$

$$(1+p_2)\frac{20p_2}{1+p_2} = (1+p_2)\,10$$

$$20p_2 = 10 + 10p_2$$

$$10p_2 = 10$$

$$p_2 = 1$$

Does this also clear the market for good 2.

$$\frac{\frac{1}{2}(20)}{1} + \frac{20(1)}{1+1} = 20$$

$$10 + 10 = 20$$

$$20 = 20$$

$p_1 = 1, p_2 = 1$ is a set of equilibrium prices.

d) What are the equilbrium allocations.

The demands are:

$$x_{1,a} = \frac{\frac{1}{2}(20(1))}{1}, x_{2,a} = \frac{\frac{1}{2}(20(1))}{1}$$

$$x_{1,b} = \frac{20(1)}{1+1}, x_{2,b} = \frac{20(1)}{1+1}$$

$$x_{1,a} = 10, x_{2,a} = 10$$

$$x_{1,b} = 10, x_{2,b} = 10$$

First welfare theorem says that every equilibrium outcome of a exchange economy (with no externalities) will be pareto efficient.

e) No, because the allocations are pareto efficient.