1 Chapter 9

$$p_1x_1 + p_2x_2 = m$$

$$p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2$$

Intercepts:

$$\left(\frac{p_1\omega_1 + p_2\omega_2}{p_1}, 0\right)$$

$$\left(0, \frac{p_1\omega_1 + p_2\omega_2}{p_2}\right)$$

Gross deamdns (how much does the consumer want)

$$(x_1, x_2)$$

Net demand (difference between the endowment and gross demand)

$$(\omega_1 - x_1, \omega_2 - x_2)$$

$\omega_1 = 5$ $\omega_2 = 5$

$x_1 = 10$ $x_2 = 0$

Gross demands are 10 and 0

Net demand 5 and -5

Net buyer of good 1 and a net seller of good 2.

If a consumer is a net buyer of a good and the price decreases, they remain and net buyer and are strictly better off.

If a consumer is a net seller of a good and the price increases, they remain and net seller and are strictly better off.

2 Chapter 10

Market / Aggregate Demand: Sum of individual demands.

$$x = 10 - p$$

The market demand is:

$$100\,(10 - p)$$

$$x = 1000 - 100p$$

Demand function has quantity (q,x,y) as a function of price.

Inverse demand is opposite, it is price as a function of quantity.

Inverse demand asks "what price would consumers pay for a certainty quantity of good"

Let's say the market demand function is:

$$q = 1000 - 100p$$

$$100p = 1000 - q$$

$$p = 10 - \frac{1}{100}q$$

2.1 Elasticities

Elasticities measure a relationship in percentage terms.

Instead of saying a one dollar increase in price leads to a 100 unit decrease in demand.

Price elasticity of demand:

We ask what is the percent change in demand for a 1% increase in price?

-1 for a 1% increase in price, there is a 1% **decrease** in demand. **Unit-Elastic**

-10 for a 1% increase in price, there is a 10% **decrease** in demand. **Elastic**

-0.1 for a 1% increase in price, there is a 0.1% **decrease** in demand. **Inelastic**

$$\epsilon_{1,1} = -1$$

Cross-price elasiticity:

We ask what is the percent change in demand for a 1% increase in price of some other good?

$\epsilon_{1,2} = -1$

The demand for good 1 goes down by 1% when the price of good 2 goes up by 1%.

$\epsilon_{1,2} = 1$

The demand for good 1 goes up by 1% when the price of good 2 goes up by 1%.

Income elasiticity:

We ask what is the percent change in demand for a 1% increase in income.

Suppose the income elasticity of a good is 1. Interpret this number.

$\eta = 1$

When income goes up by 1% demand goes up by 1%

$\eta = -1$

When income goes up by 1% demand goes down by 1%

2.2 Calculating Elasticity:

$x_1 = \frac{\frac{1}{2}m}{p_1}$

Price elasticity:

$$\epsilon_{1,1} = \frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial p_1}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}} = \frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial p_1}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}}$$

$$\frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial p_1} = -1\frac{1}{2}mp_1^{-2} = \frac{-\frac{1}{2}m}{p_1^2}$$

$$\frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial p_1}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}} = \frac{-\frac{1}{2}m}{p_1^2}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}} = -\frac{1}{p_1^2}\frac{p_1}{\frac{1}{p_1}} = -\frac{p_1}{p_1} = -1$$

Income elasticity:

$$\frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial m}\frac{m}{\frac{\frac{1}{2}m}{p_1}} = \frac{\frac{1}{2}}{p_1}\frac{m}{\frac{\frac{1}{2}m}{p_1}}$$

$$= \frac{\frac{1}{2}}{p_1} \frac{m}{\frac{\frac{1}{2}m}{p_1}} = \frac{\frac{1}{2}m}{\frac{1}{2}m} = 1$$

3 Chapter 11

Demand: $q_d = 2000 - 30p$

Supply: $q_s = 10p$

Equilibrium is determined by a price where supply and demand are the same.

$$2000 - 30p = 10p$$

$$2000 = 40p$$

$$p = 50$$

$$q = 500$$

Inverse demand and supply:

Demand: $q_d = 2000 - 30p$

Supply: $q_s = 10p$

Demand: $p = \frac{2000}{30} - \frac{1}{30}q_d$

Supply: $p = \frac{1}{10}q_s$

Without a tax:

$$2000 - 30p = 10p$$

With a tax. Treat p as the "stricker price" that suppliers get.

$$2000 - 30\left(p + t\right) = 10p$$

If $t = 10$

$$2000 - 30\left(p + 10\right) = 10p$$

Suppliers get (in equlibrium)

$$p = 42.5$$

Consumer pay (in equilibrium):

$$p + t = 52.5$$

4 Chapter 12

Production functions.

Isoquantis are sets of input bundles tht produce the same output (analogous to indifference curves)

$$f(x_1, x_2) = 3x_1 + 2x_2$$

$(2, 0)$ and $(0, 3)$ are on the same isoquant

two bundles that both produce 6 units of output

Slope of an isoquant measures how much of x_2 firm can give up if they use one more unit of x_1 (for producing the same amount of output). This is measured by the **technical rate of substitution:**

$$-\frac{mp_1}{mp_2} = -\frac{\frac{\partial(f(x_1,x_2))}{\partial x_1}}{\frac{\partial(f(x_1,x_2))}{\partial x_2}}$$

In the example:

$$f(x_1, x_2) = 3x_1 + 2x_2$$

$$TRS = -\frac{mp_1}{mp_2} = -\frac{3}{2}$$

$f(x_1, x_2) = 3x_1 + 2x_2$

$mp_1 = 3$

$f(x_1, x_2) = x_1^{\frac{1}{2}} + x_1^{\frac{1}{2}}$

$mp_1 = \frac{\partial\left(x_1^{\frac{1}{2}} + x_1^{\frac{1}{2}}\right)}{\partial x_1} = \frac{1}{2}x_1^{-\frac{1}{2}} = \frac{1}{2}\frac{1}{x_1^{\frac{1}{2}}}$

Decreasing marginal product (less extra output for additonal input) .

Returns to scale. How does output change when out scale up both inputs.

$$f(x_1, x_2) = 3x_1 + 2x_2$$

$$f(1,1), f(2,2)$$

$$f(1,1) = 5, f(2,2) = 10$$

Doubled the inputs and also doubled the output. Linear returns to scale.

$$f(x_1, x_2) = x_1 x_2$$

$$f(1,1), f(2,2)$$

$$f(1,1) = 1, f(2,2) = 4$$

Doubling inputs more than doubels outputs. **increasing returns to scale.**

$$f(1,1) = 10, f(2,2) = 15$$

Decreasing returns to scale.

5 Chapter 13

Cost minimization

Find the bundle that is cheapest and produces a desired amount of output.

If you can take derivatives of $f(x_1, x_2)$ look for where:

$$trs = -\frac{w_1}{w_2}$$

$$f(x_1, x_2) = y$$

For example:

$w_1 = 1, w_2 = 1$

$$f(x_1, x_2) = x_1 x_2$$

What bundle minimizes the cost of producing 4 units of output?

The bundle has to meet the production constraint:

$$x_1 x_2 = 4$$

Tangency condition:

$$-\frac{x_2}{x_1} = -1$$

$$x_2 = x_1$$

Plug this into the production constraint:

$$x_1 x_1 = 4$$

$$x_1^2 = 4$$

$$x_1 = 2$$

$$x_2 = 2$$

$$(2, 2)$$

5.1 Perfect complements:

$$min\{x_1, x_2\} = 4$$

$$x_1 = x_2$$

$$min\{x_1, x_1\} = 4$$

$$x_1 = 4$$

$$x_2 = 4$$

5.2 Perfect Subs

$$f(x_1, x_2) = 3x_1 + 2x_2$$

$w_1 = 1, w_2 = 1$

What is the cheapest way to produce 6?

Just figure out wheter it is cheaper to use all x_1 or all x_2 to produce output.

$$(2, 0), (0, 3)$$

The cost of these are:

$$2, 3$$

Clearly, $(2, 0)$ is the cheapest way of producing output.

6 Chapter 14

Price-taking

$c(q)$

Find the quantity that maximizes the firms profit.

$$\pi = rev - cost$$

$c(q) = q^2$, $p = 10$

$$\pi = 10q - q^2$$

$$\frac{\partial \left(10q - q^2\right)}{\partial q} = 0?$$

$$10 - 2q = 0$$

$$q = 5$$

$$p = 10$$

7 Chapter 15

Monopoly

$c(q) = q^2$

Inverse demand: $p = 100 - q$

$$\pi = (100 - q)\,q - q^2$$

$$= 100q - 2q^2$$

The quantity that maximizes profit is the one where marginal profit is zero (slope is zero):

$$\frac{\partial\left(100q - 2q^2\right)}{\partial q} = 0$$

$$100 - 4q = 0$$

$$q^* = 25$$

Plug into the inverse demand to get optimal price:

$$p^* = (100 - 25) = 75$$

8 Chapter 16

Types of price discrimination.

First: Charge Everyone their full willingness to pay.

Second: Make different options available and let consumers choose for themselves.

Third: Identify different groups and charge them different amounts.

Bundling: Sell two different types of goods in a bundle and charge one price rather than selling the goods separately.

Two part tarrif: Charge and entry-fee or up-front fee for the right to buy goods at a lower unit price.

See homework for types of problems.

9 Chapter 17

Cournot *two* firms q_1, q_2

$$\pi_1 = (100 - (q_1 + q_2))\, q_1 - q_1^2$$

This is maximized for any q_2 where:

$$\frac{\partial \left((100 - (q_1 + q_2))\, q_1 - q_1^2 \right)}{\partial q_1} = 0$$

$$-4q_1 - q_2 + 100 = 0$$

Solve for q_1 to get the best response:

$$q_1 = \frac{1}{4}(100 - q_2)$$

$$q_2 = \frac{1}{4}(100 - q_1)$$

To find a symmetric equilibrium:

$$q = \frac{1}{4}(100 - q)$$

$$4q = 100$$

$$q^* = 20$$

Total market quantity is 40. Price

$$p^* = (100 - 40) = 60$$