1 Chapter 9

$$p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2$$

Endpoints:

$$\left(\frac{p_1\omega_1 + p_2\omega_2}{p_1}, 0\right), \left(0, \frac{p_1\omega_1 + p_2\omega_2}{p_2}\right)$$

Gross and net demand:

Gross demand is how much the consumer wants:

$\omega_1 = 5, \omega_2 = 5$

Gross demand: $x_1 = 10, x_2 = 0$

Net demand: (difference between endowment and gross demand):

$$(x_1 - \omega_1, x_2 - \omega_2)$$

$$(5, -5)$$

The consumer is a net buyer of good 1 and a net seller of good 2.

If the consumer is a net buyer of a good and the price of that good decreases, they will remain a net buyer and be strictly better off.

If the consumer is a net seller of a good and the price of that good increases, they will remain a net seller and be strictly better off.

2 Chapter 10

2.1 Market Demand

Market demand is the sum of individual demands.

Individual demands:

$$q = 10 - p$$

Market demand is the sum of individual demands.

Suppose we have 100 consumers with this demand. The market demand is:

$$100\,(10-p)$$

If we have three consumers with demands:

$$q_1 = 10 - p, q_2 = 10 - 2p, q_3 = 10 - 3p$$

$$10 - p + 10 - 2p + 10 - 3p = 30 - 6p$$

Inverse demand solving for p in a demand function.

$$q = 30 - 6p$$

$$p = 5 - \frac{1}{6}q$$

The most that consumer will pay for quantity q.

2.2 Elasticities

An elasticity measures a relationship between two things in percentage terms.

Price Elasticity:

$$\epsilon_{1,1} = -1$$

If the price of good 1 goes up by 1% demand goes **down** by 1%. **Unit Elastic**

$$\epsilon_{1,1} = -10$$

If the price of good 1 goes up by 1% demand goes **down** by 10%. **Elastic Demand.**

$$\epsilon_{1,1} = -0.1$$

If the price of good 1 goes up by 1% demand goes **down** by 0.1%. **Inelastic Demand.**

Cross-price elasiticity:

$$\epsilon_{1,2} = -1$$

Supose the cross price elasticity for demand of good 1 against the price of good 2 is

$$\epsilon_{1,2} = -1$$

If the price of good 2 goes up by 1% demand for good one goes down by 1%.

$$\epsilon_{1,2} = 2$$

If the price of good 2 goes up by 1% demand for good one goes up by 2%.

Income elasiticity:

$$\eta = 1$$

If income goes up by 1% demand for good one goes up by 1%.

$$\eta = -10$$

If income goes up by 1% demand for good one goes down by 10%.

2.2.1 Calculating Elasticity:

$x_1 = \frac{\frac{1}{2}m}{p_1}$
Price elasticity:

$$\frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial p_1}\frac{p_1}{\left(\frac{\frac{1}{2}m}{p_1}\right)} = -\frac{m}{2p_1^2}\frac{p_1}{\left(\frac{\frac{1}{2}m}{p_1}\right)} = -\frac{mp_1^2}{2p_1^2\frac{1}{2}m} = -1$$

Cross Price elasticity:

$$\frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial p_2}\frac{p_2}{\frac{\frac{1}{2}m}{p_1}} = 0$$

Income elasticity:

$$\frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial m}\frac{m}{\left(\frac{\frac{1}{2}m}{p_1}\right)} = \frac{\frac{1}{2}}{p_1}\frac{m}{\frac{\frac{1}{2}m}{p_1}} = 1$$

3 Chapter 11

$q_d = 81 - p$

$q_s = 8p$

$$81 - p = 8p$$

$$81 = 9p$$

$$p^* = 9$$

$$q^* = 72$$

Inverse demand:

$$q = 81 - p$$

$$p = 81 - q$$

Inverse supply:

$$p = \frac{1}{8}q$$

Without a tax:

$$81 - (p + t) = 8p$$

Suppose $t = 2$

Supplier gets in equilirbium:

$$p^* = 8.77778$$

Consumer pays:

$$p^* + 2 = 10.7778$$

4 Chapter 12

$$f(x_1, x_2)$$

What bundle produce the same amount of output.

$$f(x_1, x_2) = x_1 x_2$$

Want to produce 4 units of output:

$$(4,1), (1,4), (2,2)$$

The bundles are all on the same **isoquant** (the isoquant tht produces 4 units of output). ,

TRS (technical rate of substitution) measures the slope of an isoquant at a point:

$$trs = -\frac{mp_1}{mp_2} = -\frac{\frac{\partial(f(x_1,x_2))}{\partial x_1}}{\frac{\partial(f(x_1,x_2))}{\partial x_2}}$$

Finding marginal products.

Determining if decreaing marginal product.

Determining the returns to scale of a production funtion.

$f(x_1, x_2) = x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}$

$$mp_1 = \frac{\partial\left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}\right)}{\partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}} = \frac{1}{2} \frac{1}{x_1^{\frac{1}{2}}}$$

Diminishing marginal product since mp_1 is decreasing in x_1.

$f(x_1, x_2) = x_1^2 + x_2^2$

$$mp_1 = 2x_1$$

Not diminishing marginal product.

Returns to scale:

$f(x_1, x_2) = x_1^2 + x_2^2$

$$f(1,1), f(2,2)$$

$$f(1,1) = 2, f(2,2) = 8$$

Increasing returns to scale.

$f(x_1, x_2) = x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}$

$$f(1,1), f(2,2)$$

$$f(1,1) = 2, f(2,2) = 2\sqrt{2.0} = 2.82843$$

Decreasing returns to scale.

$f(x_1, x_2) = x_1 + x_2$

$$f(1,1), f(2,2)$$

$$f(1,1) = 2, f(2,2) = 4$$

Linear returns to scale.

5 Chapter 13

5.1 Production function with well-defined technical rate of substitution:

$f(x_1, x_2) = x_1 x_2$, $w_1 = 1, w_2 = 1$, $y = 4$

$$-\frac{x_2}{x_1} = -\frac{1}{1}$$

$$x_1 x_2 = y$$

Equal slope condition simplies to:

$$x_1 = x_2$$

Plug this into the production constraint:

$$x_1 x_1 = y$$

$$x_1^2 = y$$

$$x_1 = \sqrt{y}$$

$$x_2 = \sqrt{y}$$

$$(\sqrt{y}, \sqrt{y})$$

Cost of producing y in the cheapest way:

$$1\sqrt{y} + 1\sqrt{y}$$

$$c(y) = 2\sqrt{y}$$

For $y = 4$

$$(2, 2)$$

5.2 Perfect Complements

$w_1 = 1, w_2 = 1, \; y = 4$

$$f(x_1, x_2) = min\{x_1, x_2\}$$

What is the cheapest way of producing output 4?

Production constraint:

$$min\{x_1, x_2\} = 4$$

$$x_1 = x_2$$

Plug the no waste condition into the production constraint:

$$min\left\{x_1, x_1\right\} = 4$$

$$x_1 = 4, x_2 = 4$$

Perfect substitutes

$f\left(x_1, x_2\right) = 3x_1 + 2x_2$

$w_1 = 1, w_2 = 1,\ y = 4$

$$-\frac{3}{2} = -\frac{1}{1}$$

Either use all x_1 or all x_2. Which is cheaper:

Use all x_1 how much do I need to produce output 4?

$$3x_1 + 2\left(0\right) = 4$$

$$x_1 = \frac{3}{4}$$

$$\left(\frac{3}{4}, 0\right)$$

Cost of this bundle is $\frac{3}{4}$

If I use all x_2

$$3\left(0\right) + 2x_2 = 4$$

$$x_2 = 2$$

$$\left(0, 2\right)$$

Cost of this bundle is 2

The cost minimizing bundle is:

$$\left(\frac{3}{4}, 0\right)$$

6 Chapter 14

Price taking

$c(q) = q^2$

$p = 10$

$$\pi = 10q - q^2$$

Profit is maximized where:

$$\frac{\partial\left(10q - q^2\right)}{\partial q} = 0$$

$$10 - 2q = 0$$

$$q = 5$$

7 Chapter 15

Monopoly

$c(q) = q^2$

Inverse demand: $p = 100 - q$

$$\pi = (100 - q)\, q - q^2$$

$$= 100q - 2q^2$$

The quantity that maximizes profit is the one where marginal profit is zero (slope is zero):

$$\frac{\partial\left(100q - 2q^2\right)}{\partial q} = 0$$

$$100 - 4q = 0$$

$$q^* = 25$$

Plug into the inverse demand to get optimal price:

$$p^* = (100 - 25) = 75$$

8 Chapter 16

Types of price discrimination.

First: Charge Everyone their full willingness to pay.

Second: Make different options available and let consumers choose for themselves.

Third: Identify different groups and charge them different amounts.

Bundling: Sell two different types of goods in a bundle and charge one price rather than selling the goods separately.

Two part tarrif: Charge and entry-fee or up-front fee for the right to buy goods at a lower unit price.

See homework for types of problems.

9 Chapter 17

Cournot *two* firms q_1, q_2

$$\pi_1 = (100 - (q_1 + q_2))\, q_1 - q_1^2$$

This is maximized for any q_2 where:

$$\frac{\partial \left((100 - (q_1 + q_2))\, q_1 - q_1^2\right)}{\partial q_1} = 0$$

$$-4q_1 - q_2 + 100 = 0$$

Solve for q_1 to get the best response:

$$q_1 = \frac{1}{4}(100 - q_2)$$

$$q_2 = \frac{1}{4}\left(100 - q_1\right)$$

To find a symmetric equilibrium:

$$q = \frac{1}{4}\left(100 - q\right)$$

$$4q = 100$$

$$q^* = 20$$

Total market quantity is 40. Price

$$p^* = (100 - 40) = 60$$