1 Calculus Review

1.1 Why calculus?

Mountains.

1.2 Notation

With a function $f(x)$ the derivative is denoted either:

$$f'(x)$$

or:

$$\frac{\partial(f(x))}{\partial x}$$

1.2.1 Power Rule

Rule: x^a the derivative is: ax^{a-1}

$f(x) = x^2$. $f'(x) = 2x$

$f(x) = \sqrt{x} = x^{\frac{1}{2}}$. $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}\frac{1}{x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

$f(x) = \frac{1}{x} = x^{-1}$. $f'(x) = -1x^{-2} = -\frac{1}{x^2}$

$f(x) = x^e$. $f'(x) = ex^{e-1}$

1.3 Derivative of a Sum

The derivative of a sum is the sum of the derivatives.

$f(x) = 2x^7 + 3x^4$. $f'(x) = 14x^6 + 12x^3$

1.4 Natural Log

The derivative of $ln(x)$ is $\frac{1}{x}$.

$f(x) = ln(x) + x^2$. $f'(x) = \frac{1}{x} + 2x$

1.5 Product Rule

The derivative of the product of two things is the derivative of the first times the second plus the derivative of the second times the first.

$f(x) = ln(x)\,x^2$. $f'(x) = \frac{1}{x}x^2 + 2xln(x)$

1.6 Chain Rule

If you have a function of a function, the derivative is the derivative of the outside function times the derivative of the inside function.

$f(x) = ln\left(x^2\right)$. $f'(x) = \frac{1}{x^2}2x = 2\frac{x}{x^2} = \frac{2}{x}$

$f(x) = ln(3x+1) = \frac{1}{3x+1}(3) = \frac{3}{3x+1}$

$f(x) = \sqrt{ln(x)} = (ln(x))^{\frac{1}{2}}$. $f'(x) = \frac{1}{2}(ln(x))^{\frac{1}{2}-1}\left(\frac{1}{x}\right) = \frac{1}{2}(ln(x))^{-\frac{1}{2}}\frac{1}{x}$

2 Partial Derivatives

How does this function change as we change x? (and hold y constant).

$$\frac{\partial f(x,y)}{\partial x}$$

How does this function change as we change y? (and hold x constant).

$$\frac{\partial f(x,y)}{\partial y}$$

$f(x,y) = x^2y^2$

$$\frac{\partial\left(x^2y^2\right)}{\partial x}?$$

Let's imagine $y = 5$

$$\frac{\partial\left(x^25^2\right)}{\partial x} = 2x5^2$$

y is unknown

$$\frac{\partial\left(x^2y^2\right)}{\partial x} = 2xy^2$$

$$\frac{\partial\left(x^2y^2\right)}{\partial y} = x^22y = 2x^2y$$

xy

$$\frac{\partial (xy)}{\partial x} = y$$

$$\frac{\partial (xy)}{\partial y} = x$$

$x^3 + y^3 + 3xy$

$$\frac{\partial \left(x^3 + y^3 + 3xy\right)}{\partial x} = 3x^2 + 0 + 3y = 3x^2 + 3y$$

$f(x, y) = x + ln(y)$

$\frac{\partial (x+ln(y))}{\partial x} = 1$

$\frac{\partial (x+ln(y))}{\partial y} = \frac{1}{y}$

Second Derivaties The second derivative is simply the derivative of the derivative.

$f(x) = x^3.\ f'(x) = 3x^2$

$f''(x) = 6x$