1 Calculus Review

1.1 Why calculus?

1.2 Notation

$f(x)$

Derivative of $f(x)$ is $f'(x)$. The slope of the function at the point x.

Both of these mean the same thing:

$$f'(x) = \frac{\partial(f(x))}{\partial x}$$

1.2.1 Power Rule

If we have a function like this $f(x) = x^{\alpha}$

The derivative is

$f(x) = x^2$. $f'(x) = 2x$

$f(x) = \sqrt{x} = x^{\frac{1}{2}}$. $f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}}$

$$= \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$f(x) = \frac{1}{x} = x^{-1}$. $f'(x) = -1x^{-1-1} = -x^{-2}$

$$= -\frac{1}{x^2}$$

$f(x) = x^e$. $f'(x) = ex^{e-1}$

1.3 Natual Log

$$f(x) = ln(x)$$

$$f'(x) = \frac{1}{x}$$

1.3.1 Derivative of a Sum

The derivative of a sum is the sum of derivatives:

$f(x) = 2x^7 + 3x^4$. $f'(x) = 14x^6 + 12x^3$

$f(x) = ln(x) + x^2$. $f'(x) = \frac{1}{x} + 2x$

1.3.2 Product Rule

The derivative of a product of two functions is the derivative of the first times the second, plus the derivative of the second times the first.

$f(x) = ln(x)\, x^2$. $f'(x) = \frac{1}{x}x^2 + (2x)\, ln(x)$

1.3.3 Chain Rule

Applies when we have a function of a function. Take the derivative of the outside and multiply it by the derivative of the inside.

$f(x) = ln\left(x^2\right)$. $f'(x) = \frac{1}{x^2}(2x)$

$f(x) = ln(3x+1)$. $f'(x) = \frac{1}{3x+1}3 = \frac{3}{3x+1}$

$f(x) = \sqrt{ln(x)} = (ln(x))^{\frac{1}{2}}$.

$$f'(x) = \left(\frac{1}{2}ln(x)^{-\frac{1}{2}}\right)\frac{1}{x}$$

2 Partial Derivatives

The slope of the function when we change the variable x? (and hold y fixed)

$$\frac{\partial f(x,y)}{\partial x}$$

The slope of the function when we change the variable y? (and hold x fixed)

$$\frac{\partial f(x,y)}{\partial y}$$

$f(x) = x^2 * 5^2$

$$f'(x) = 2x * 5^2$$

$f(x,y) = x^2y^2$

$$\frac{\partial\left(x^2y^2\right)}{\partial x} = 2xy^2$$

$$\frac{\partial\left(x^2y^2\right)}{\partial y} = 2yx^2$$

$f(x,y) = xy$

$$\frac{\partial(xy)}{\partial x} = y$$

$$\frac{\partial(xy)}{\partial y} = x$$

$f(x,y) = x^3 + y^3 + 3xy$

$$= x^3 + x^0y^3 + 3xy$$

$$\frac{\partial\left(x^3 + y^3 + 3xy\right)}{\partial x} = 3x^2 + 0 + 3y = 3x^2 + 3y$$

$$= 3x^2 + 3y$$

$f(x,y) = ln(x) + y$

$$\frac{\partial(ln(x) + y)}{\partial x} = \frac{1}{x}$$

$$\frac{\partial(ln(x) + y)}{\partial y} = 1$$

Second Derivaties The second derivative of a function is the derivative of the derivative.

$f(x) = x^3$. $f'(x) = 3x^2$

$$f''(x) = 6x$$