1 Binary Relations

Binary Relation is a mathematical concept that describes relationships between pairs of things in a set.

$\geq$ is a relation **on the set of numbers**.

$$3 \geq 2$$

A relation can refer to an element of a set and iteself. That is true here.

$$3 \geq 3$$

$>$ is also a relation on the set of numbers.

$$3 > 2$$

Here the relation is not true of an element of the set and itself:

$$3 \not> 3$$

At least as tall as is a relation on the set of all people. Let use R for it.

Since Shaq (s) is taller than Greg (g), we write:

$$\text{sRg}$$

Since Greg is not taller than Shaq, we write:

$$g \not R s$$

Sibling of is a relation on the set of people.

Greg is a sibling of Christina

$$gRc$$

Christina is a sibling of Greg

$$cRg$$

The relation is **symmetric** since if it goes one direction, it always goes the other.

An asymmetric relation is one where if it goes one direction ,it **never** goes the other. Strictly taller than is **assymetric.** It can only every be true in one direction.

2 Properties

Three properties we really care about in economics.

2.1 Reflexive

Reflexive: Everything is related to itself in that way.

$>, \geq$,at least as tall as, strictly taller than.

Of these, $\geq$ and **at least as tall as** are **reflexive**

Same biological parents as is reflexive.

2.2 Complete

Complete: Every pair of things including a thing and itself are related in at least one direction.

Same biological parents as: **Not Complete**

At least as tall as: **Complete**

Strictly taller than: **Not complete**

$\geq$ **Complete**

$>$ Not complete since it is not reflexive.

A complete relation must be reflexive!

2.3 Transitive

Transitive: For every three things a, b, c. If aRb and bRc then it must be that aRc.

At least as tall as: T**ransitive**

Beat in a tennis match: **Not Transitive**

Beats in the set of actions in rock paper scissors: **Not Transitive**

Paper beats rock. rock beats scissors, but paper does not beat scissors!

3 Preference Relation

A preference relation is a binary relation on the fesible set X.

$\succsim$

If one bundle is "prefered" to another, we write:

$$(2, 0) \succsim (1, 0)$$

This says: "Two scoops of vanilla with zero chocolate is "at least as good as" to one scoop of vanilla and zero chocolate."

$$(2,2) \succsim (1,1)$$

"Two scoops of each flavor is at least as good as one scoop of each flavor."

3.1 Rational Preferences

A preference relation needs to meet two conditions to be useful for doing economics. It must be **complete** and must be **transitive**. If it meets these conditions, we say it is "**Rational**"

Complete: they can compare any pair of things that is relevant in the model. **Transitivity**: their preferences put things in a ordering.

These two assumptions ensure that from any set of things, the consumer will be able to choose.

3.2 Choice and "Best"

From a set B we say that a bundle is **Best** if it is at least good as all other bundles in the set.

$a \succsim b, b \succsim c, a \succsim c, a \succsim a, b \succsim b, c \succsim c$

What is best from the set $\{a, b, c\}$? a is "best".

We assume that the choice a consumer will make is the thing that is best from the set. We use the c hoice function to indicate what is best.

$$C(\{a, b, c\}) = a$$

This says "a" is best from the set $\{a, b, c\}$.

$$C(\{a, b\}) = a$$

$$C(\{b, c\}) = b$$

Another example:

$a \succsim b, a \succsim c, a \succsim d, b \succsim a, b \succsim c, b \succsim d, c \succsim d,$

$a \succsim a, b \succsim b, c \succsim c, d \succsim d$

$$C(\{a, b, c, d\}) = \{a, b\}$$

$$C(\{b,c,d\}) = \{b\}$$

$$C(\{c,d\}) = \{c\}$$

$$C(\{a,d\}) = \{a\}$$

3.3 Why Complete and Transitive

$a \succsim b, b \succsim c, a \succsim a, b \succsim b, c \succsim c$: **Not Complete**

$$C(\{a,c\}) = \emptyset$$

The consumer can't choose from $\{a,c\}$!

$a \succsim b, b \succsim c, c \succsim a, a \succsim a, b \succsim b, c \succsim c$: **Not Transitive**

$$C(\{a,b,c\}) = \emptyset$$

The consumer can't choose from $\{a,b,c\}$!

As long as preferences are complete and transitive, then the choice set will be non-empty for every set of things.

4 Strict Preference and Indifference

Suppose we have:

$$(2,0) \succsim (1,0)$$

$$(1,0) \not\succsim (2,0)$$

When a relation is true in only one direction, we write:

$$(2,0) \succ (1,0)$$

And we say $(2,0)$is **strictly preferred** to $(1,0)$.

Similarly, if $(2,2) \succsim (1,1), (1,1) \not\succsim (2,2)$ then $(2,2) \succ (1,1)$

Suppose we have:

$$(1,0) \succsim (0,1)$$

$$(0,1) \succsim (1,0)$$

When the preference relation goes in both directions, we write:

$$(1,0) \sim (0,1)$$

We say $(1,0)$ and $(0,1)$ are **indifferent.**

5 Chain Notation:

$a \succsim b, a \succsim c, a \succsim d, b \succsim a, b \succsim c, b \succsim d, c \succsim d,$

$a \succsim a, b \succsim b, c \succsim c, d \succsim d$

Chain notation, takes the weak preferences and creates the ordering using only indifference and strict preference. Better things are on the left.

$$a \sim b \succ c \succ d$$

This summarizes all the information in one simple notation that is easy to read. When preferences are complete and transitive, we can always put the preferences into a chain notation.