1 Budget Lines

B the set of bundles a consumer can afford.

The cost of a bundle if the prices of the goods are p_1 for x_1 and p_2 for x_2.

Cost of bundle (x_1, x_2):

$$p_1x_1 + p_2x_2$$

A bundle is **affordable** if (income is m).

$$p_1x_1 + p_2x_2 \leq m$$

This inequality is what we sometimes call the **budget set.**

The budget line is the set of bundles that are "just affordable". The cost of the bundle is equal to m.

$$p_1x_1 + p_2x_2 = m$$

Suppose $p_1 = 2, p_2 = 1, m = 10$:

$$2x_1 + 1x_2 = 10$$

1.1 Endpoints of Budget Line

The amount of x_1 the consumer can afford if they only buy x_1:

$$\frac{m}{p_1}$$

In the case of $p_1 = 2, p_2 = 1, m = 10$

$$\frac{m}{p_1} = \frac{10}{2} = 5$$

The amount of x_2 the consumer can afford if they only buy x_2:

$$\frac{m}{p_2}$$

$$\frac{m}{p_2} = \frac{10}{1} = 10$$

1.2 Slope of Budget Line

The Slope of Budget Line always reresents how much of the good on the vertical axis (x_2) do I have to give up to get one more unit of the thing on the horizontal axis (x_1)?

$$-\frac{p_1}{p_2}$$

For instance, if $p_1 = 2, p_2 = 1$.

1.3 How Budget Line Changes

Did this on the board.

1.4 Taxes

Taxes simply increase the price of a good.

1.4.1 Quantity Tax

An amount of extra money t owed per unit of the good purchased.

If we have a **quantitiy** tax t on good x_1.

The new cost of bundle (x_1, x_2)

$$tx_1 + p_1x_1 + p_2x_2$$

New budget line:

$$tx_1 + p_1x_1 + p_2x_2 = m$$

$$(p_1 + t)\, x_1 + p_2x_2 = m$$

This effectively changes the price from p_1 to $p_1 + t$.

Example: gasoline.

1.4.2 Ad Valorem Tax

An Ad Valorem Tax is a percentage of the total cost of a good purchased.

If we have an Ad Valorem Tax τ on good 1 :

$$\tau (p_1 x_1) + p_1 x_1 + p_2 x_2$$

New budget line:

$$(1 + \tau) p_1 x_1 + p_2 x_2 = m$$

x_1 now becomes τ percentage more expensive than it was.

2 Binary Relations

We use binary relations to represent preferences in economics.

I like a scoop of vanilla better than a scoop of chocolate.

$$(1, 0) \succsim (0, 1)$$

2.1 What is a Relation?

A relation is an abstract mathematica way of expressing relationships betwen **pairs of things in a set**.

2.2 Examples from Everyday Life

The set of all people X.

Suppose R is the Sibling relation.

Greg. $g \in X$. Christina $c \in X$

"Greg is a sibling of christina"

$$gRc$$

"Christina is a sibling of greg"

$$cRg$$

"Greg is not a sibling of Shaq"

$$g \not{R} s$$

Suppose R is "At least as tall as".

Greg is at least as tall as Christina

$$gRc$$

$$c \not{R} g$$

$$gRg$$

$$sRg$$

$$g \not{R} s$$

"Strictly taller than"

$$gRc$$

$$g \not{R} g$$

2.3 Examples from Math

$=, <, \leq, \geq, >$ are all relations on the set of numbers $\mathbb{R}$

$$3 > 2$$

2.4 Properties

2.4.1 Reflexive

We say a relation on the set X is **reflexive** if for every element of X:

$$xRx$$

Are Reflexive On the set of all people:

Same Height As.

Same biological parents as.

At least as tall as.

Math:

$=, \geq, \leq$, "shares a common factor"

Are Not Reflexive On the set of all people:

Strictly taller than.

Strictly older than.

Math:

$>, <$

2.4.2 Complete

We say a relation R on the set X is **complete** if for every pair of elements in X (including x and itself).

Either xRy or yRx or both.

On The Set of All People

Same Biological Parents as? **No.**

$$g \not R s, s \not R g$$

At least as tall as? **Complete.**

On the set of all numbers?

$=$, **No.** $3 \neq 2, 2 \neq 3$

$\geq$, **Yes**

$>$, **No** $2 \ngtr 2, 2 \ngtr 2$

If a relation is not reflexive, then it is not complete.