1 Relations

R is the sibling relationship.

gRc, cRg

$s\not R g, g \not R s$

1.1 Complete

For every pair of things, some relationship holds.

A relation is on a set X.

$>$ is a relation on the set of numbers $\mathbb{R}$

A relation on the set X is complete if for every pair of elements from $x_1, x_2 \in X$ including an element and itself $x_1 = x_2$.

Either $x_1 R x_2$ or $x_2 R x_1$ or both.

If you can a pair such that neither statement is true, the relation is **not complete.**

Complete Relations:

$\geq$ is complete on the set of all number.

At least as all tall as on the of all people.

Incomplete Relations:

$>$ is incomplete. It is not true that $5 > 5$

In order for a relation to be complete, it also has to be **reflexive**.

Sibling on the set of all people is incomplete. For instance, Greg (g) and Shaq (s) neither gRs nor sRg.

1.2 Transitive

There is a natural "ordering" to the relation.

A relation is transitive if for every three elements, $x_1, x_2, x_3 \in X$

If $x_1 R x_2$ **and** $x_2 R x_3$ then $x_1 R x_3$**.**

Examples of transitive relations.

$>$ $3 > 2$ and $2 > 1$

$\geq$

At least as tall as is transitive.

Same hair color.

Same biological parents as is transitive.

Won in a tennis match against is **intransitive.**

Friendship is intransitive.

fRn and nRd and dRf

1.3 Symmetric

If the relationship holds in one direction, it holds in both directions.

A relation is symmetric if for every pair $x_1, x_2 \in X$

If $x_1 R x_2$ then $x_2 R x_1$.

Sibling is symmetric.

Same hair color symmetric.

Same height is symmetric.

Married to is symmetric.

Strcitly taller than is not symmetric. In fact it is **asymetric.**

1.4 Asymmetric

If it holds in one direction, **it does not hold in the other direction**.

Strictly taller than is asymetric.

Is a parent of is asymetric.

$>$ on the set of numbers is asymetric.

1.5 Exercise 2.4

$\{x, y, z\}$

$R : xRy, yRz, xRz$

Is this complete? No, it is not reflexive.

Is this transitive?

xRy, yRz transitivity implies that xRz. This checks out.

$R : xRy, yRz, zRx$

Is this transitive?

xRy, yRz however we don't have xRz. Thus, this is intransitive.

$R : xRx, yRy, zRz$

This one is what we call **vacuously** true.

1.6 Another Example

$R : xRy, yRz, zRx, xRz$

Is this transitive.

xRy, yRz we need to check whether xRz. It does.

zRx, xRy we need to check whether zRy. But it isn't.

Intransitive.

zRx, xRz is zRz? No.

1.7 Is every relexive relationship symmetric?

$\{x, y, z\}$

$xRx, yRy, zRz, xRy, yRz, xRz$

Another Example $R : xRy, yRz, xRz, xRx, yRy, zRz$

Is complete.

2 Preference Relations

We use relations in economics to represent preferences.

2.1 Weak Preference Relation

"The bundle x_1 is at least good as x_2."

$$x_1 \succsim x_2$$

"x_1 is prefered to x_2."

"x_1 is weakly prefered to x_2."

If you offer them x_1 and x_2 they would be perfectly happy to have x_1.

With just the weak preference relation, we can also express indifference and strictly preference.

2.2 Indifference

If i'm truly indifference between x_1 and x_2, then if you offered me both, I would be happy to have either.

$$x_1 \succsim x_2, x_2 \succsim x_1$$

The consumer is **indifferent** between x_1 and x_2.

$$x_1 \sim x_2$$

We say $x_1 \sim x_2$ if $x_1 \succsim x_2$ and $x_2 \succsim x_1$.

Indifference is **induced** by weak prefernce. If I know your weak preference relation, I also know your indifference relation.

2.3 Strict Preference

$$x_1 \succsim x_2, x_2 \not\succsim x_1$$

x_1 must be strictly prefereed to x_2.

Preferences are strict when:

$x_1 \succ x_2$ when $x_1 \succsim x_2, x_2 \not\succsim x_1$.

2.4 Example.

$X = \{a, b, c\}$

$a \succsim a, b \succsim b, c \succsim c,$

$a \succsim b, b \succsim a, a \succsim c, b \succsim c$

Do I like apples strictly more than bananas or am I indifferent?

$a \sim a, b \sim b, c \sim c$

$a \sim b, a \succ c, b \succ c$

What would they choose from the set $\{b, c\}$? b

$a \succsim a, b \succsim b, c \succsim c,$

$a \succsim b, b \succsim a, a \succsim c, b \succsim c$

b is the **best** element of the set $\{b, c\}$

$\{a, c\}$? a

a is the **best** element of the set $\{a, c\}$

$\{a, b, c\}$? a or b

a, b are the **best** elements of the set $\{a, b, c\}$

We an element of a set is **best** if it as at least as good as **every other element in the set**.

2.4.1 Rational Preferences

$a \succsim a, b \succsim b, c \succsim c, a \succsim b$

This is incomplete. There is no statement about preferences between b, c.

What is the best element of:

$$\{b, c\}$$

There is no element that is at least as good as every other element. **There is no best element.**

Another example:

$a \succsim a, b \succsim b, c \succsim c$

$a \succsim b, b \succsim c, c \succsim a$

$\{a, b\} : a$

$\{b, c\} : b$

$\{a, c\} : c$

$\{a, b, c\}$: $\emptyset$

$a \succ b, b \succ c, c \succ a$

If preference are complete and transitive can I always make a choice from any menu?

Yes.

As long as the menu does not have an infinite number of things in it, if prefeerence are complete and transtiive, there is always some best element (best elements).

2.5 Rational

If preference are complete and transitive, we say they are **Rational.**