1 Relations

A relation represents relationships between pairs of things in a set X.

1.1 Complete

A relation is **complete** if every pair (including an element and itself) of objects has some relationship.

For every $x, y \in X$

Either xRy or yRx or both.

Examples:

$X = \{a, b, c, d, ..., z\}$. R represents "comes at least as early in the alphabet".

Is R complete? Yes, for every of letters one is at least as early in the alphabet as the other.

Is $\geq$ on the set of numbers complete?

For every pair of numbers one has to be at least as big as the other, and so this is complete.

$>$ is not complete. For example for the pair 5 and 5 there is no true statement involving $>$.

R is the "cousin of" relation on the set of all people.

$$g \not R c, c \not R g$$

$$g \not R g$$

If you can find a pair where neither direction hold. Or a single thing where the relation doesn't hold for that thing and itself. **The relation is not complete.**

If for every pair there is some direction that holds and for every individual thing, that relation is true for that thing and itself, **the relation is complete**.

For a relation to be complete, it has to be reflexive.

"At least as tall as" on the set of all people. Is this complete? This is complete.

"strictly taller than" is not complete.

If there are two people the same height, then neither direction holds. Or, for a person and themselves the relation doesn't hold.

1.2 Transitive

A relation is transitive if for every three things, $x, y, z \in X$

If xRy and yRz then xRz.

Height is transitive.

If one person is taller than another and they are taller than a third person, the first is also taller than the third.

"At least as tall as" is transitive.

"Strictly taller than" is transitive.

$>, \geq, =$ are all transitive.

$3 > 2, 2 > 1$ $3 > 1$

Non-Examples:

"beats in a tennis match"

Friendship. Not transitive since a and b could be friends and b and c could be friends, but a and c do not know eachother.

$aRb, bRc, a\not{R}c$

1.3 Extended 2.4,3

xRy, yRz, zRx, xRz is this transitive?

1.3.1 Symmetric

If one direction holds, then both hold.

For every pair $x, y \in X$ if xRy then yRx.

Same biological parents. Symmetric.

$=$ on the set of numbers. Symmetric.

$>$ is not symmetric. In fact, this is asymetric.

$3 > 2$

1.3.2 Asymmetric

For every pair elements, both directions never hold.

For every pair $x, y \in X$ if xRy then $y\not{R}x$.

$>$

strictly taller than

1.3.3 Exercise 2.4

$\{x, y, z\}$

$R : xRy, yRz, xRz$

$R : xRy, yRz, zRx$

$R : xRx, yRy, zRz$

2 Preference Relations

If I like one scoop of vanilla $(1, 0)$ at least as much as one scoop of chocolate $(0, 1)$ then we write:

2.1 Weak Preference Relation

$$(1, 0) \succsim (0, 1)$$

$(1, 0)$ is at least good as $(0, 1)$

$(1, 0)$ is weakly prefered to $(0, 1)$

If you gave me the option between $(1, 0)$ and $(0, 1)$, I would be happy to have $(1, 0)$.

Finn like more ice cream rather than less. Otherwise, for two bowls that have the same amount, he like the one with more vanilla.

$$(2, 0) \succsim (1, 0)\,,$$

$$(1, 0) \succsim (0, 1)$$

$$(2, 1) \succsim (1, 2)$$

2.2 Indifference

If I told you from $(1, 0)$ $(0, 1)$ I would be happy to have either.

$$(1, 0) \succsim (0, 1)$$

$$(0,1) \succsim (1,0)$$

You would infer that I am indifferent.

If for a pair of bundles, $x \succsim y$ and $y \succsim x$ we say "x and y are indifferent" and we write:

$$x \sim y$$

2.3 Strict Preference

If I told you from $(1,0)$ $(0,1)$ I would be happy to have $(1,0)$ but not $(0,1)$

$$(1,0) \succsim (0,1)$$

$$(0,1) \not\succsim (1,0)$$

You would infer that I like $(1,0)$ strictly more thant $(0,1)$

If for a pair of bundles, if $x \succsim y$ and $y \not\succsim x$ we say "x is strictly prefered to y" and we write:

$$x \succ y$$

2.4 Example.

$X = \{a, b, c\}$

$a \succsim a, b \succsim b, c \succsim c,$

$a \succsim b, b \succsim a, a \succsim c, b \succsim c$

$a \sim b, a \succ c, b \succ c$

$\{a, b, c\}$, they would happy to have either a, b

$\{a, c\}$, a

$\{b, c\}$, b

$\{a, b\}$, a, b

2.5 Best

A object from a set is "best" according to a preference relation if it is at least good as everything in the set.

$a \succsim a, b \succsim b, c \succsim c,$

$a \succsim b, b \succsim a, a \succsim c, b \succsim c$

$\{a, b, c\}$

a is best from this set.

b is best from this set.

2.6 Rational

If a preference relation is complete and transitive, we say it is **rational**.

Suppose completeness fails.

$a \succsim a, b \succsim b, c \succsim c, a \succsim c$

$$\{a, b\}$$

There is no best element.

$a \succsim a, b \succsim b, c \succsim c, a \succsim b, b \succsim c, c \succsim a$

Intransitive.

$$\{a, b, c\}$$

There is no best element.

2.7 Why Rational

If preferences are complete and transitive, for every (finnite) set, there is at least one best element.