1 Prelude

An intransitive preference relation creates a "cycle" of strict preferences:

$a \succsim b, b \succsim c, c \succsim a$

$a \succsim a, b \succsim b, c \succsim c$

Let's find the strict preference relation: $\succ$

$a \succ b, b \succ c, c \succ a$

What is the choice from a,b,c? There can't be one!

$$C(\{a, b, c\}) = \emptyset$$

2 Utility Functions

Rational. *Complete and Transitive*

$$a \succ b \sim c \succ d$$

$a : 4, b : 3, c : 3, d : 2$

$a : 10, b : 1, c : 1, d : 0$

Utility function. A way of assigning numbers to the bundles such that those numbers are consistent with the underlying preference relation.

$$b \succ a \sim d \succ c$$

$u(a) = 5, u(b) = 7, u(c) = 2, u(d) = 5$

$u(a) = 6, u(b) = 8, u(c) = 3, u(d) = 6$

$u(a) = 10, u(b) = 14, u(c) = 4, u(d) = 10$

There are many utility functions that represent the same preferences.

$u(a) = 2, u(b) = 1, u(c) = 6, u(d) = 6$

$$c \sim d \succ a \succ b$$

2.1 Perfect Substitutes

Finn's preference for ice cream is that he only cares about total scoops. x_1 is vanilla and x_2 is chocolate

$$(3,2) \succ (2,2)$$

$$(3,2) \sim (2,3)$$

$u(x_1, x_2) = x_1 + x_2$

$$u(3,2) = 5, u(2,3) = 5, u(2,2) = 4$$

This utility function represents those preferences.

$u(x_1, x_2) = 2(x_1 + x_2) + 1$

$$u(3,2) = 2 * 5 + 1 = 11, u(2,3) = 11, u(2,2) = 9$$

Perfect substitutes preferences.

$u(x_1, x_2) = x_1 + x_2$

What bundles give finn a utility of 10?

$$x_1 + x_2 = 10$$

This is now an equation for an indiffernce curve.

$$x_2 = 10 - x_1$$

Perfect substitues prefernces can be represented by utility functions of this form:

$$u(x_1, x_2) = ax_1 + bx_2$$

2.2 MRS From Utility

MRS is the slope of the utility function.

How do we get the MRS at a point from a utility function.

Marginal Utility:

$$MU_1 = \frac{\partial(u(x_1, x_2))}{\partial x_1}$$

How much does utility increase if I get a little more x_1?

$$MU_2 = \frac{\partial\left(u\left(x_1, x_2\right)\right)}{\partial x_2}$$

How much does utility increase if I get a little more x_2?

$$MRS = -\frac{MU_1}{MU_2}$$

$$-\frac{\frac{\partial(x_1+x_2)}{\partial x_1}}{\frac{\partial(x_1+x_2)}{\partial x_2}} = -\frac{1}{1} = -1$$

2.3 Cobb Douglass

$$x_1^{\alpha} x_2^{\beta}$$

$$u\left(x_1, x_2\right) = x_1 x_2$$

$MU_1 = \frac{\partial(x_1 x_2)}{\partial x_1} = x_2$

$MU_2 = \frac{\partial(x_1 x_2)}{\partial_2} = x_1$

$$MRS = -\frac{x_2}{x_1}$$

3 Exercise

$$\sqrt{x_1} + x_2$$

$$(9, 4)$$

$$u\left(9, 4\right) = \sqrt{9} + 4 = 3 + 4 = 7$$

$$u\left(0, x_2\right) = x_2$$

$$u\left(0, x_2\right) = 7$$

$$x_2 = 7$$

3.1 Exercise 7

$$x_1 x_2$$

$$u(9,4) = 9 * 4 = 36$$

$$u(1,1) = 1$$

$$u(2,2) = 4$$

$$u(3,3) = 9$$

$$u(z,z) = z^2$$

$$z^2 = 36$$

$$z = 6$$

3.2 Exercise 8

1) $3x_1 + 2x_2$

$mu_1 = \frac{\partial(3x_1+2x_2)}{\partial x_1} = 3$

$mu_2 = \frac{\partial(3x_1+2x_2)}{\partial x_2} = 2$

$$MRS = -\frac{3}{2}$$

2) $x_1 x_2$

$$MRS = -\frac{x_2}{x_1}$$

At $(2,2)$ the MRS is:

$$-\frac{2}{2} = -1$$

3) $4x_1x_2 + 10$

$mu_1 = \frac{\partial(4x_1x_2+10)}{\partial x_1} = 4x_2$

$mu_2 = \frac{\partial(4x_1x_2+10)}{\partial x_2} = 4x_1$

$$MRS = -\frac{x_2}{x_1}$$

At $(2, 2)$ the MRS is:

$$-\frac{2}{2} = -1$$

4) $ln\,(x_1) + x_2$

$mu_1 = \frac{1}{x_1}$

$mu_2 = 1$

$$MRS = -\frac{\frac{1}{x_1}}{1} = -\frac{1}{x_1}$$

at $(2, 2)$

$$MRS = -\frac{1}{2}$$

5) $x_1 + x_1x_2$

$$mu_1 = 1 + x_2$$

$$mu_2 = 0 + x_1 = x_1$$

$$MRS = -\frac{1 + x_2}{x_1}$$

At $(2, 2)$

$$MRS = -\frac{3}{2}$$