1 Convexity

1.1 Convex Combinations

$$(2,0),(0,2)$$

$(1,1)$ is a an average/mixtures of these bundles

$(1.5,0.5)$

$(0.5,1.5)$

Convex Combinations generalize the mixtures. Gives us all the ways of mixing two bundles together.

$t \in [0,1]$ the weight put on bundle 1.

$(1-t)$ the weight on bundle 2.

$t=\frac{1}{2}$ puts equal weight on both. For $(2,0),(0,2)$ results in $(1,1)$

$t=\frac{3}{4}$ puts $\frac{3}{4}$ on bundle 1 and $\frac{1}{4}$ weight on bundle 2. Results in $(1.5,0.5)$.

(x_1,x_2) and (y_1,y_2)

The weighted average good 1: $tx_1+(1-t)\,y_1$

The weighted average good 2: $tx_2+(1-t)\,y_2$

$$(tx_1+(1-t)\,y_1, tx_2+(1-t)\,y_2)$$

1.2 Convex Preferences

Any mixture of two bundles you are indifference between is at least as good as the two original bundles.

If $(x_1,x_2)\sim(y_1,y_2)$

$$(tx_1+(1-t)\,y_1, tx_2+(1-t)\,y_2)\succsim(x_1,x_2)$$

$$(tx_1+(1-t)\,y_1, tx_2+(1-t)\,y_2)\succsim(y_1,y_2)$$

Mixtures are at least as good as extremes.

1.3 Examples

$Min\{x_1, x_2\}$ is convex.

$Min\{x_1, x_2\}$ is not convex.

(Examples on the board)

Perfect Complements, Perfect Substitutes, Cobb-Douglass, Quasi-Linear preferences are all **convex.**

1.3.1 Geometry of Convex Preferences

A line between two points on any indifference curve will lie on or above the indifference curve.

1.4 Well-Behaved

If preferences are rational, **and monotonic and convex.** We say they are **well-behaved.**

Perfect Complements, Perfect Substitutes, Cobb-Douglass, Quasi-Linear preferences are all **well-behaved.**

2 Constrained Optimization

2.1 Unconstrained Optimization: Flat at the Top

With unconstrained optimization, we just need to find a point where the slope of the function is zero in all directions.

$$100 - (x_1 - 10)^2 - (x_2 - 10)^2$$

At the peak, both of these need to be true:

$$\frac{\partial \left(100 - (x_1 - 10)^2 - (x_2 - 10)^2\right)}{\partial x_1} = 0$$

$$\frac{\partial \left(100 - (x_1 - 10)^2 - (x_2 - 10)^2\right)}{\partial x_2} = 0$$

Simplifying these, both of these need to be true at the peak:

$$-2\left(x_1 - 10\right) = 0, -2\left(x_2 - 10\right) = 0$$

$$x_1 = 10, x_2 = 10$$

2.2 3 Possibilities for an Optimal Bundle

If preferences are monotonic, the optimal bundle has to be **on the budget line.**

A point on the budget line that is on an indifference curve that crosses into the interior of the budget set **can never be optimal.**

Since this is the case,

Any optimal bundle will be on an indifference that **just touches but does not cross into the interior of the budet set**.

1. The indifference is **tanget** to the budget line. (The slope of the indifference curve and budget line are the same).

2. The optimal bundle is on a "kink" on the indifference curve that just touches the budget line.

3. It is at a boundary where either $x_1 = 0$ or $x_2 = 0$

2.3 Intuition for Case #1

1. The indifference is **tanget** to the budget line. (The slope of the indifference curve and budget line are the same).

$$MRS = -\frac{p_1}{p_2}$$

$$-\frac{mu_1}{mu_2} = -\frac{p_1}{p_2}$$

Suppose it wasn't true:

$$\frac{mu_1}{mu_2} > \frac{p_1}{p_2}$$

I am willing to give up more x_2 than I have to get more x_1.

2.4 Another Intuition

$$-\frac{mu_1}{mu_2} = -\frac{p_1}{p_2}$$

$$\frac{mu_1}{p_1} = \frac{mu_2}{p_2}$$

Bang-for-buck $\frac{mu_1}{p_1}$ is how much utility goes up when you spend one more dollar on good 1.

If these aren't equal, I should take money away from the good tht gives me less utility per dollar and put it in to the good that gives me more.

Cobb Douglass Example $p_1 = 2, p_2 = 1, m = 20$ and $u(x_1, x_2) = x_1 x_2$

What bundle (x_1, x_2) maximizes utility subject to this budget constraint.

Since we can find the MRS of $u(x_1, x_2)$, start with condition 1.

$$MRS = -\frac{p_1}{p_2}$$

$$-\frac{\frac{\partial(x_1 x_2)}{\partial x_1}}{\frac{\partial(x_1 x_2)}{\partial x_2}} = -\frac{2}{1}$$

$$-\frac{x_2}{x_1} = -\frac{2}{1}$$

Tangency Condition:

$$x_2 = 2x_1$$

Budget Condition:

$$2x_1 + x_2 = 20$$

Two conditions and two unknowns:

$$2x_1 + x_2 = 20$$

Plug in the tangency condition:

$$2x_1 + (2x_1) = 20$$

$$4x_1 = 20$$

$$x_1 = 5$$

Plug this back into the tangency condition:

$$x_2 = 2x_1$$

$$x_2 = 2(5)$$

$$x_2 = 10$$

$$(5, 10)$$

Perfect Substitutes Example $p_1 = 2, p_2 = 1, m = 20$ and $u(x_1, x_2) = x_1 + x_2$

MRS=$-\frac{1}{1} = -1$

$$-1 = -\frac{2}{1}$$

$$-1 = -2$$

If this happens, you know one the intercepts will be optimal.

$$\left(\frac{m}{p_1}, 0\right), \left(0, \frac{m}{p_2}\right)$$

Which is these give me higher utility?

$$(10, 0), (0, 20)$$

Which gives more utility?

$$u(10, 0) = 10, u(0, 20) = 20$$

$$(0, 20)$$