1 Convexity

1.1 Convex Combinations

$$(2,0),(0,2)$$

One way to average these is the "normal" average.

$$(1,1)$$

Other ways to "average" or mix these bowls are to take $t=\frac{3}{4}$ weight on the first bundle and $1-t=\frac{1}{4}$ on the second bundle.

$$\left(\frac{3}{4}2+\frac{1}{4}0,\frac{3}{4}0+\frac{1}{4}2\right)$$

$$(1.5,0.5)$$

All of the convex combinations (mixtures) of these bowls can be acheived this way:

$$(x_1,x_2),(y_1,y_2)$$

t weight on bundle 1 and $(1-t)$ weight on bundle 2

$$(tx_1+(1-t)\,y_1,tx_2+(1-t)\,y_2)$$

1.2 Convex Preferences

Preferences are **convex** if any convex combination (mixture) of two bundles that indifferent between has to be at least as good as the two original bundles.

$$(x_1,x_2)\sim(y_1,y_2)$$

$$(tx_1+(1-t)\,y_1,tx_2+(1-t)\,y_2)\succsim(x_1,x_2)$$

$$(tx_1+(1-t)\,y_1,tx_2+(1-t)\,y_2)\succsim(y_1,y_2)$$

This implies that mixtures cannot be worse than the two bundles being mixed.

1.2.1 Geometry of Convex Preferences

Preferences are convex if for **any two points on any indifference curve, a line between those two points lies on or above the indifference curve.**

1.3 Well-Behaved

We say prefrences are "well-behaved" when they are rational (complete and transitive) **and** monotonic + convex.

2 Constrained Optimization

2.1 Unconstrained Optimization: Flat at the Top

$$100 - (x_1 - 10)^2 - (x_2 - 10)^2$$

Where is this maximized? What is the (x_1, x_2)

First-Order Conditions

$$\frac{\partial \left(100 - (x_1 - 10)^2 - (x_2 - 10)^2\right)}{\partial x_1} = 0$$

$$\frac{\partial \left(100 - (x_1 - 10)^2 - (x_2 - 10)^2\right)}{\partial x_2} = 0$$

Solve these:

$$-2\left(x_1 - 10\right) = 0$$

$$-2\left(x_2 - 10\right) = 0$$

What is the slope of the mountain at the coordinates $(10, 0)$?

$$\left(-2\left(10 - 10\right), -2\left(0 - 10\right)\right)$$

$$(0, 20)$$

This is not the peak. The slope is zero in the x_1 direction, but not the x_2 direction.

To find the peak, solve this system of equations:

$$-2(x_1 - 10) = 0$$

$$x_1 - 10 = 0$$

$$x_1 = 10$$

$$-2(x_2 - 10) = 0$$

$$x_2 = 10$$

The only solution to the first order condition:

$$(10, 10)$$

2.2 3 Possibilities for an Optimal Bundle

If preferences are monotonic, the optimal point has to be **on the budget line** (the boundry of the budget set).

Universal Truth:

The optimal bundle has to on the budget and the indifference through that point has to just touch but not cross into the budget set.

1. If the indifference curve has well-defined slope, it is the same as the slope of the budget line. $MRS = -\frac{p_1}{p_2}$

2. If the indifference curves have kinks then the optimal point has to occur at a kink.

3. The slope of the indifference curve is never the same as the slope of the budget line, then one of the endpoints of the budget will be optimal.

2.3 Intuition for Case #1

2.4 Another Intuition

2.4.1 Cobb Douglass Example

$p_1 = 2, p_2 = 1, m = 20$ and $u(x_1, x_2) = x_1 x_2$

What bundle maximizes the consumer's utility, given their budget?

Equal-Slope / Tangency Condition:

$$MRS = -\frac{p_1}{p_2}$$

$$-\frac{\frac{\partial(x_1x_2)}{\partial x_1}}{\frac{\partial(x_1x_2)}{\partial x_2}} = -\frac{2}{1}$$

$$-\frac{x_2}{x_1} = -\frac{2}{1}$$

$$\frac{x_2}{x_1} = 2$$

$$\frac{x_2}{x_1}x_1 = 2x_1$$

$$x_2 = 2x_1$$

Equal-Slope Condition.

$$x_2 = 2x_1$$

Budget-Condition:

$$2x_1 + x_2 = 20$$

Plug the equal-slope condition into the budget line:

$$2x_1 + 2x_1 = 20$$

$$4x_1 = 20$$

$$x_1 = 5$$

Plug this back into the equal-slope condition:

$$x_2 = 2\,(5)$$

$$x_2 = 10$$

$$(5, 10)$$

2.4.2 Perfect Substitutes Example

$p_1 = 2, p_2 = 1, m = 20$ and $u\,(x_1, x_2) = x_1 + x_2$