

# 1 Chapter 8

Slutsky Decomposition

## 1.1 Two Effects

**Income effect.** When the price of something goes up, your income is effectively worth less. You might change your consumption based on that.

**Substitution effect.** When the price of something goes up, you will substitute into buying other things to satisfy your preferences and so you will buy less.

## 2 Law of Demand

**The substitution effect is always negative.**

The income effect can be positive or negative depending on whether the good is **normal** or **inferior**.

**A Giffen Good** is an inferior good where where when the price of a good goes up, the income effect is so strong that leads to an increase in **demand**.

Three possibilities:

Ordinary, Normal. (Starbucks coffee)

Ordinary, Inferior.

Giffen, Inferior.

## 3 Slutsky Decomposition

How do we determine mathematically how much of the change in demand for a good is due to the substitution effect and how much is due to the income effect?

Suppose the price of the good changes, but I give you enough money at the new price to buy exactly the bundle you were buying before.

Suppose you were buying (10, 10) but the price of  $x_1$  went up and your demand changes to (5, 15). And now I give you extra money so you could still afford (10, 10) at the new prices, but instead you buy (7, 12).

## 4 Slutsky Decomposition Problem

Exercise 3.

$$u(x_1, x_2) = x_1 x_2$$

$p_1 = 100, p_2 = 100$  and  $m = 1200$

Find the marshallian demand for  $x_1, x_2$ :

Budget Condition:

$$p_1x_1 + p_2x_2 = m$$

Equal-slope condition:

$$MRS = -\frac{p_1}{p_2}$$

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

Two equations and two unknowns. Let's solve them.

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

Let's get rid of the negatives by multiplying both sides by  $-1$ .

$$\frac{x_2}{x_1} = \frac{p_1}{p_2}$$

Multiply both sides by  $x_1$

$$x_1 \frac{x_2}{x_1} = x_1 \frac{p_1}{p_2}$$

$$x_2 = x_1 \frac{p_1}{p_2}$$

Plug this into the budget equation:

$$p_1x_1 + p_2 \left( x_1 \frac{p_1}{p_2} \right) = m$$

$$p_1x_1 + p_1x_1 = m$$

$$2p_1x_1 = m$$

Divide both sides by  $2p_1$

$$x_1 = \frac{m}{2p_1}$$

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

Plug this back into the equal slope condition:

$$x_2 = x_1 \frac{p_1}{p_2}$$

$$x_2 = \frac{\frac{1}{2}m}{p_1} \frac{p_1}{p_2}$$

The  $p_1$  cancel:

$$x_2 = \frac{\frac{1}{2}m}{p_2}$$

$$\left( \frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2} \right)$$

$p_1 = 100, p_2 = 100$  and  $m = 1200$

a)  $\frac{\frac{1}{2}1200}{100}, \frac{\frac{1}{2}1200}{100}$

$$(6, 6)$$

b)  $\frac{\frac{1}{2}1200}{200}, \frac{\frac{1}{2}1200}{100}$

$$(3, 6)$$

Total change in demand is  $-3$ .

c) What is the price of the old bundle at the new prices?

$$(200 * 6) + (100 * 6) = 1800$$

d) What do they buy at  $p_1 = 200, p_2 = 100, m = 1800$ ?

$$\left( \frac{\frac{1}{2}1800}{200}, \frac{\frac{1}{2}1800}{100} \right)$$

$$(4.5, 9)$$

e) The substitution effect is  $-1.5$  (difference between original 6 of  $x_1$  and 4.5 of  $x_1$ )

f) Income effect is the remainder. Total effect was  $-3$ . Substitution effect is  $-1.5$  of that. So, income effect is  $-1.5$ .