1 Finding Demand

1. Indifference is tangent to the budget line at the optimal bundle.

$$MRS = -\frac{p_1}{p_2}$$

2. Indifference curve is just touching but not tangent because the slope isn't well-defined. (Optimal point is at the kink).

3. The slope of at indifference curve is never the same as the slope of the budget line. One of the two endpoints will always be optimal.

1.1 Perfect Complements

$min\left\{x_1, x_2\right\}, p_1 = 1, p_2 = 2, m = 30$

The optimal point for a perfect complements utility function always occurs at the kink.

No-Waste Condition:

$$x_1 = x_2$$

Budget Condition:

$$x_1 + 2x_2 = 30$$

Plug in the no-waste condition into the budget line:

$$x_1 + 2x_2 = 30$$

$$(x_2) + 2x_2 = 30$$

$$3x_2 = 30$$

$$x_2 = 10$$

Plug this back into the no waste condition:

$$x_1 = x_2$$

$$x_1 = 10$$

$$(10, 10)$$

$min \left\{x_1, \frac{1}{2}x_2\right\}, p_1 = 3, p_2 = 1, m = 30$
No waste:

$$x_1 = \frac{1}{2}x_2$$

$$2x_1 = x_2$$

Budget:

$$3x_1 + x_2 = 30$$

$$3x_1 + (2x_1) = 30$$

$$5x_1 = 30$$

$$x_1 = 6$$

$$x_2 = 2x_1$$

$$x_2 = 2\,(6) = 12$$

$$(6, 12)$$

Redux:

$$x_1 = \frac{1}{2}x_2$$

$$3\left(\frac{1}{2}x_2\right)+x_2=30$$

$$\frac{3}{2}x_2+x_2=30$$

$$\frac{5}{2}x_2=30$$

$$x_2=\frac{2}{5}30$$

$$x_2=12$$

1.2 Quasi-Linear

$ln\left(x_1\right)+x_2, p_1=10, p_2=2, m=30$

Tangency:

$$MRS=-\frac{p_1}{p_2}$$

$$-\frac{\frac{\partial(ln(x_1)+x_2)}{\partial x_1}}{\frac{\partial(ln(x_1)+x_2)}{\partial x_2}}=-\frac{10}{2}$$

$$\frac{\partial\left(ln\left(x_1\right)+x_2\right)}{\partial x_1}=\frac{1}{x_1}$$

$$\frac{\partial\left(ln\left(x_1\right)+x_2\right)}{\partial x_2}=1$$

$$-\frac{\frac{1}{x_1}}{1}=-\frac{10}{2}$$

$$-\frac{1}{x_1} = -5$$

$$x_1 = \frac{1}{5}$$

For a quasi-linear problem, the tangency condition will immediately give you the optimal amoun of the non-linear good (x_1 here). To the get optimal amount of the other good, use the budget constraint.

$$10x_1 + 2x_2 = 30$$

Plug the optimal amount of x_1 into this:

$$10\left(\frac{1}{5}\right) + 2x_2 = 30$$

$$2 + 2x_2 = 30$$

$$2x_2 = 28$$

$$x_2 = 14$$

$$\left(\frac{1}{5}, 14\right)$$

1.3 Perfect Substitutes

$x_1 + x_2, p_1 = 1, p_2 = 2, m = 30$

Tangency condition:

$$-\frac{1}{1} = -\frac{1}{2}$$

The optimal bundle has to be at one of the "boundaries" that is, at one of the end-points of the budget constraint.

$$\left(\frac{m}{p_1}, 0\right), \left(0, \frac{m}{p_2}\right)$$

Check the utility of these two bundles. Pick the better one.

$$(30, 0), (0, 15)$$

$$u(30, 0) = 30, u(0, 15) = 15$$

$$(30, 0)$$

1.4 Perfect Substitutes

$x_1 + 4x_2, p_1 = 1, p_2 = 2, m = 30$

$$(30, 0), (0, 15)$$

$$u(30, 0) = 30, u(0, 15) = 60$$

$$(0, 15)$$

1.5 Perfect Substitutes

$x_1 + 2x_2, p_1 = 1, p_2 = 2, m = 30$

$$(30, 0), (0, 15)$$

$$u(30, 0) = 30, u(0, 15) = 30$$

Any point on the budget constraint is optimal.

2 Demand

2.1 Marshallian Demand

Marshallian demand is a function that tells us how much of a good the consumer demands at **any** set of prices and income.

$$x_1^*\left(p_1, p_2, m\right), x_2^*\left(p_1, p_2, m\right)$$

2.1.1 Example: Cobb Douglass

Suppose a consumer's utility is $U\left(x_1, x_2\right) = x_1 x_2$. What is their Marshallian demand.

$$MRS = -\frac{p_1}{p_2}$$

$mu_1 = \frac{\partial(x_1 x_2)}{\partial x_1} = x_2, mu_2 = \frac{\partial(x_1 x_2)}{\partial x_2} = x_1$
Tangency condition:

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

$$\frac{x_2}{x_1} = \frac{p_1}{p_2}$$

$$x_2 = x_1 \frac{p_1}{p_2}$$

$$p_2 x_2 = p_1 x_1$$

At the optimum, the amount of money spent on both goods is the same. This utility function leads to the consumer to budget their income.

$$p_1 x_1 + p_2 x_2 = m$$

$$p_1 x_1 + \left(p_1 x_1\right) = m$$

$$2 p_1 x_1 = m$$

$$x_1 = \frac{m}{2p_1} = \frac{\frac{1}{2}m}{p_1}$$

Plug this back into the tangecy condition:

$$p_2x_2 = p_1\frac{\frac{1}{2}m}{p_1}$$

$$p_2x_2 = \frac{1}{2}m$$

$$x_2 = \frac{\frac{1}{2}m}{p_2}$$

$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2}\right)$$

Redux:

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

$$\frac{x_2}{x_1} = \frac{p_1}{p_2}$$

$$x_2 = x_1\frac{p_1}{p_2}$$

$$p_1x_1 + p_2\left(x_1\frac{p_1}{p_2}\right) = m$$

$$p_1x_1 + p_1x_1 = m$$

$$2p_1x_1 = m$$

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

The marshallian demands for $U(x_1, x_2) = x_1 x_2$ are:

$$x_1^* (p_1, p_2, m) = \frac{\frac{1}{2}m}{p_1}$$

$$x_2^* (p_1, p_2, m) = \frac{\frac{1}{2}m}{p_2}$$

2.1.2 Example: Perfect Comlements

$U(x_1, x_2) = min \{x_1, x_2\}$

$$x_1 = \frac{m}{p_1 + p_2}, x_2 = \frac{m}{p_1 + p_2}$$

No-waste condition:

$$x_1 = x_2$$

Budget condition:

$$p_1 x_1 + p_2 x_2 = m$$

Plug tangency into budget:

$$p_1 (x_2) + p_2 x_2 = m$$

$$p_1 x_2 + p_2 x_2 = m$$

$$x_2 (p_1 + p_2) = m$$

$$x_2 = \frac{m}{p_1 + p_2}$$

Plug this back into no-waste condition:

$$\frac{m}{p_1 + p_2} = x_1$$