1 Finding Demand

1. Tangency. $MRS = -\frac{p_1}{p_2}$
2. Indifference curve is just touching the budget line but there is not slope of the indifference curve. (Perfect complements).
3. The slope of the indifferences are never the same as the slope of the budget line, then the optimal bundle occurs at one of the end points of the budget line. (Perfect subsitutes).

1.1 Intuition for Tangency Condition

$MRS = -\frac{p_1}{p_2}$: *willingness to trade-off between the goods is the same as the way the budget line forces you to trade off.*

If that wasn't true, either you are willing to give up more x_2 than you have to get one more unit of x_1.

Or, you are willing to give up more x_1 than you have to get one more unit of x_2.

In either case, there is some direction I can more and myself better off.

1.2 Another Intution.

$$MRS = -\frac{p_1}{p_2}$$

$$-\frac{mu_1}{mu_2} = -\frac{p_1}{p_2}$$

$$\frac{mu_1}{p_1} = \frac{mu_2}{p_2}$$

Suppose $mu_1 = 2$ and $p_1 = 1$. Spending \$1 on good one will get me 2 extra "points" of utility.

Suppose $mu_1 = 2$ and $p_1 = 2$. Spending \$1 on good one will get me 1 extra "points" of utility.

$\frac{mu_1}{p_1}$ how much extra utility do I get by spending \$1 more on x_1

$\frac{mu_2}{p_2}$ how much extra utility do I get by spending \$1 more on x_2

Bang-for-your-buck

Suppose this wasn't true:

$$\frac{mu_1}{p_1} > \frac{mu_2}{p_2}$$

I should take away a dollar from x_2 and put it into x_1 and my utility will go up.

$$\frac{mu_1}{p_1} < \frac{mu_2}{p_2}$$

I should take away a dollar from x_1 and put it into x_2 and my utility will go up.

1.2.1 Perfect Complements

$min\left\{x_1, x_2\right\}, p_1 = 1, p_2 = 2, m = 30$

At the optimum, the indifference curve has to be just touching the budget line.

There is no reason to waste money on extra ingredients.

No-Waste Condition:

$$x_1 = x_2$$

Budget Condition:

$$x_1 + 2x_2 = 30$$

Solve these equations for x_1 and x_2. Plug the first condition into the second:

$$x_1 + 2\left(x_1\right) = 30$$

$$3x_1 = 30$$

$$x_1 = 10$$

Plug this back into the no waste condition:

$$x_1 = x_2$$

$$x_2 = 10$$

$$(10, 10)$$

$min\left\{\frac{1}{2}x_1, x_2\right\}, p_1 = 1, p_2 = 2, m = 40$

No-Waste Condition

Make the two things inside the min equal.

$$\frac{1}{2}x_1 = x_2$$

Budget Condition

Plug the prices and income into $p_1x_1 + p_2x_2 = m$:

$$x_1 + 2x_2 = 40$$

Plug the no-waste condition into the budget equation:

$$x_1 + 2\left(\frac{1}{2}x_1\right) = 40$$

$$x_1 + x_1 = 40$$

$$2x_1 = 40$$

$$x_1 = 20$$

Plug this back into the no waste condition:

$$\frac{1}{2}x_1 = x_2$$

$$\frac{1}{2}(20) = x_2$$

$$10 = x_2$$

$$(20, 10)$$

Quasi-Linear $ln\left(x_1\right)+x_2, p_1=10, p_2=2, m=30$

Tangency Condition:

$$MRS=-\frac{p_1}{p_2}$$

$mu_1=\frac{\partial(ln(x_1)+x_2)}{\partial x_1}=\frac{1}{x_1}$

$mu_2=\frac{\partial(ln(x_1)+x_2)}{\partial x_2}=1$

$$MRS=-\frac{\frac{1}{x_1}}{1}=-\frac{1}{x_1}$$

Tangency Condition:

$$-\frac{1}{x_1}=-\frac{10}{2}$$

$$\frac{1}{x_1}=5$$

$$x_1=\frac{1}{5}$$

Plug this into the budget condition to get x_2. How much x_2 can I afford if buy $\frac{1}{5}$ x_1?

$$p_1=10, p_2=2, m=30$$

$$10x_1+2x_2=30$$

Plug in $x_1=\frac{1}{5}$:

$$10\frac{1}{5}+2x_2=30$$

$$2+2x_2=30$$

$$2x_2 = 28$$

$$x_2 = 14$$

$$\left(\frac{1}{5}, 14\right)$$

Try this one at home if you want:

$\sqrt{x_1} + x_2, p_1 = 10, p_2 = 2, m = 30$

1.3 Cobb Douglass

$u\left(x_1, x_2\right) = x_1 x_2, p_1 = 1, p_2 = 2, m = 40$

Find the tangency condition:

$mu_1 = \frac{\partial (x_1 x_2)}{\partial x_1} = x_2$

$mu_2 = \frac{\partial (x_1 x_2)}{\partial x_2} = x_1$

$MRS = -\frac{x_2}{x_1}$

Tangency:

$$-\frac{x_2}{x_1} = -\frac{1}{2}$$

Budget Constraint:

$$1x_1 + 2x_2 = 40$$

Let's isolate one of the variables in the tangency condition then plug it into the budget.

$$-\frac{x_2}{x_1} = -\frac{1}{2}$$

$$\frac{x_2}{x_1} = \frac{1}{2}$$

$$x_2 = \frac{1}{2}x_1$$

Plug this into the budget

$$1x_1 + 2\left(\frac{1}{2}x_1\right) = 40$$

$$x_1 + x_1 = 40$$

$$2x_1 = 40$$

$$x_1 = 20$$

Plug this back into tangency to get x_2.

$$x_2 = \frac{1}{2}(20)$$

$$x_2 = 10$$

$$(20, 10)$$

1.4 Perfect Substitutes

$x_1 + x_2, p_1 = 1, p_2 = 2, m = 30$

Tangency:

$$-\frac{1}{1} = -\frac{1}{2}$$

The two endpoints of the budget constraint:

$$\left(\frac{m}{p_1}, 0\right), \left(0, \frac{m}{p_2}\right)$$

$$(30, 0), (0, 15)$$

The utility of these:

$$U(30, 0) = 30, U(0, 15) = 15$$

$$(30, 0)$$

1.5 Perfect Substitutes

$x_1 + 4x_2, p_1 = 1, p_2 = 2, m = 30$

$$(30, 0), (0, 15)$$

$$U(30, 0) = 30, U(0, 15) = 60$$

$$(0, 15)$$

1.6 Perfect Substitutes

$x_1 + 2x_2, p_1 = 1, p_2 = 2, m = 30$

$$(30, 0), (0, 15)$$

$$U(30, 0) = 30, U(0, 15) = 30$$

Indifferent between any bundle on the budget line.

1.6.1 Demand

Marshallian Demand

Example: Cobb Douglass Suppose a consumer's utility is $U(x_1, x_2) = x_1x_2$. What is their Marshallian demand.

Example: Perfect Comlements $U(x_1, x_2) = min\{x_1, x_2\}$