

# 1 Demand

Demand is the amount of good someone chooses at  $p_1, p_2, m$

$$x_1(1, 1, 20) = (10, 10)$$

## 1.1 Marshallian Demand

What will demand be for any set of prices and income.

**Marshallian Demand**  $x_1^*(p_1, p_2, m)$

With perfect complements preferences  $\min\{x_1, x_2\}$

$$x_1^*(p_1, p_2, m) = \frac{m}{p_1 + p_2}$$

$$x_2^*(p_1, p_2, m) = \frac{m}{p_1 + p_2}$$

### 1.1.1 Example: Cobb Douglass

$$U(x_1, x_2) = x_1 x_2$$

**Tangency Condition:**

$$MRS = -\frac{p_1}{p_2}$$

$$-\frac{\frac{\partial(x_1 x_2)}{\partial x_1}}{\frac{\partial(x_1 x_2)}{\partial x_2}} = -\frac{p_1}{p_2}$$

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

$$\frac{x_2}{x_1} = \frac{p_1}{p_2}$$

$$p_2 x_2 = p_1 x_1$$

$$x_2 = \frac{p_1}{p_2} x_1$$

**Budget Constraint:**

$$p_1x_1 + p_2x_2 = m$$

Plug in the simplified tangency condition into the budget equation:

$$x_2 = \frac{p_1}{p_2}x_1$$

$$p_1x_1 + p_2\left(\frac{p_1}{p_2}x_1\right) = m$$

$$p_1x_1 + p_1x_1 = m$$

$$2p_1x_1 = m$$

$$x_1 = \frac{m}{2p_1}$$

Plug this back into the tangency condition:

$$x_2 = \frac{p_1}{p_2}x_1$$

$$x_2 = \frac{p_1}{p_2}\left(\frac{m}{2p_1}\right) = \frac{m}{2p_2}$$

$$\left(\frac{m}{2p_1}, \frac{m}{2p_2}\right)$$

$$\left(\frac{\frac{1}{2}m}{p_1}, \frac{\frac{1}{2}m}{p_2}\right)$$

If  $m = 20, p_1 = 1, p_2 = 1$

$$\left(\frac{20}{2 * 1}, \frac{20}{2 * 1}\right)$$

$$(10, 10)$$

## 1.2 Another way to solve it

$$p_1x_1 = p_2x_2$$

$$p_1x_1 + p_2x_2 = m$$

Plug tangency into the budget constraint:

$$p_1x_1 + p_1x_1 = m$$

$$2p_1x_1 = m$$

$$x_1 = \frac{m}{2p_1}$$

## 1.3 Example: Perfect Complements

$$U(x_1, x_2) = \min\{x_1, x_2\}$$

**No-Waste Condition:**

$$x_1 = x_2$$

**Budget Constraint:**

$$p_1x_1 + p_2x_2 = m$$

Eliminate  $x_2$  by plugging the no-waste condition into the budget constraint:

$$p_1x_1 + p_2x_1 = m$$

$$x_1(p_1 + p_2) = m$$

$$x_1 = \frac{m}{p_1 + p_2}$$

Plug this into the no-waste condition:

$$x_2 = \frac{m}{p_1 + p_2}$$

$x_1$  says, buy a number of pairs of left shoes equal to the number of shoes that  $m$  can afford you.

## 1.4 Another Perfect Complements

$$\min \left\{ \frac{1}{2}x_1, x_2 \right\}$$

$$x_1 = 2 \frac{m}{2p_1 + p_2}$$

$$x_2 = \frac{m}{2p_1 + p_2}$$

## 1.5 Changes in Income

Cobb Douglass Demand:  $\frac{m}{2p_1}$

Perfect Complements Demand:  $\frac{m}{p_1 + p_2}$

### 1.5.1 Categorization

In both of our examples, as  $m$  goes up, demand goes up as well.

When this happens, we call the good an **normal** good.  $m$  goes up, demand goes up.

**Inferior good.**  $m$  goes up, demand goes down.

**Inferior** Goods: ikea furniture, instant ramen, cheap beer, replica luxury, processed food

**Normal** Goods: brand names, fresh produce, takeout food,

### 1.5.2 Not Always Inferior

A good cannot be **always** inferior.

If income is zero, demand has to be zero.

For demand to go down it has to have, at some point, gone up.

While a good can be **always normal** it can't be **always inferior**.

### 1.5.3 Engel Curve

The engel curve is a graph of how demand for a good changes as income changes.

For some specified prices, the **engel** curve is a a plot of  $x_i$  against income  $m$ .  $m$  is on the vertical axis.

Cobb Douglass Demand:  $\frac{m}{2p_1}$

$$x_1 = \frac{m}{2p_1}$$

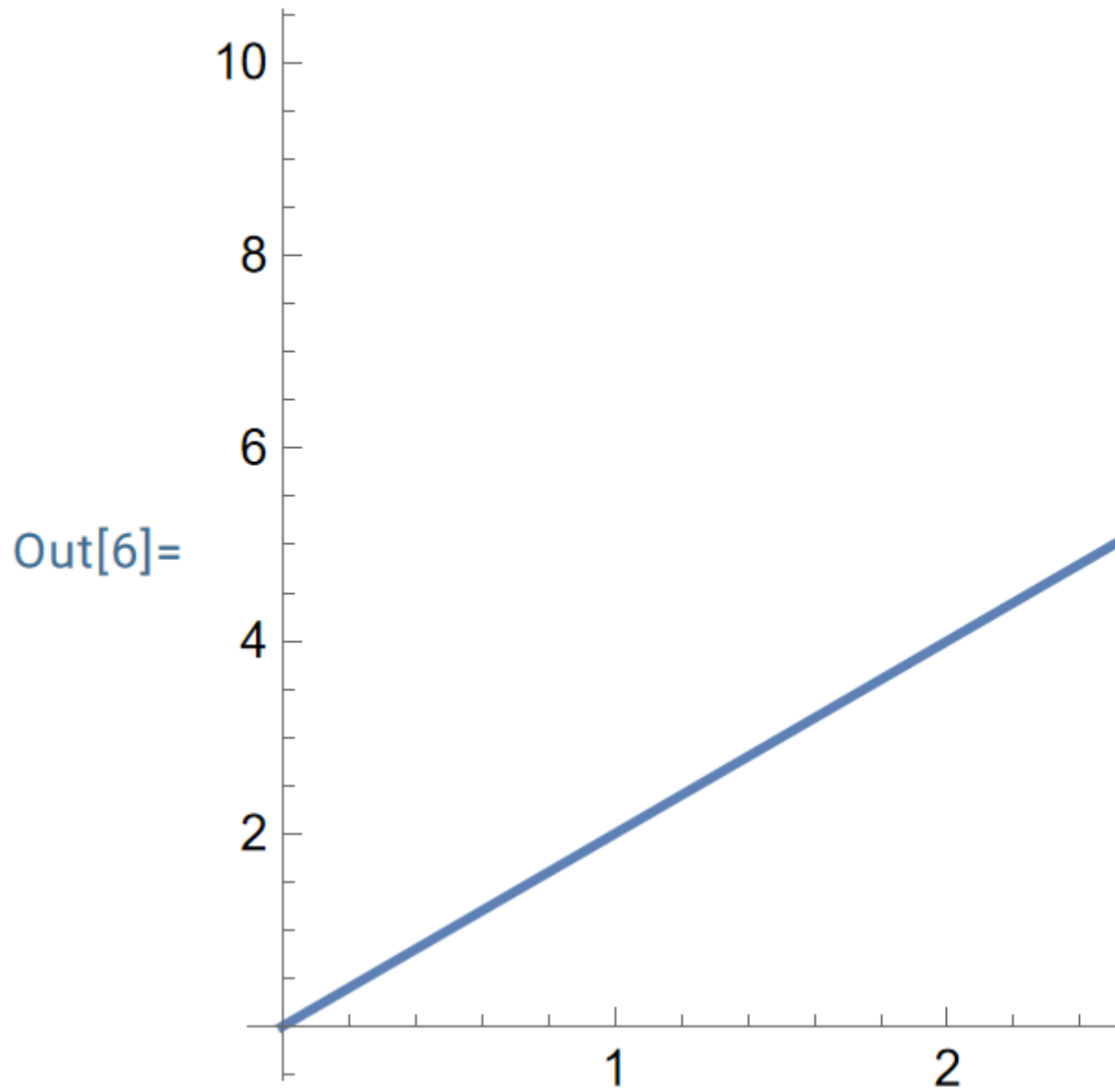
Isolate income:

$$m = 2p_1x_1$$

Suppose  $p_1 = 1, p_2 = 1$ . Plot the engel curve for  $x_1$ .

$$m = 2x_1$$

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In[6]:= Plot[2 x, {x, 0, 5}]
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Perfect Complements Demand:  $\frac{m}{p_1 + p_2}$

$$x_1 = \frac{m}{p_1 + p_2}$$

Plug in the prices  $p_1 = 1, p_2 = 1$

$$x_1 = \frac{m}{2}$$

$$2x_1 = m$$

Engle curve is a line with slope of 2.

The interpretation of the slope of the Engel curve is “how much more money do you need to give me to buy one more unit of this good”.

If the engle curve is upward sloping, the good is normal, if it bends backwards, it is inferior.

## 1.6 Changes in Own Price

How does demand change then the price of the good changes?

## 1.7 Categorization

We say a good is **ordinary** if as the price of the good goes up, demand goes down.

We say a good is **giffen** if as the price of the good goes up, demand goes up.

A giffen good has to be inferior. It has to be **really** inferior.

### 1.7.1 Inverse Demand

Suppose income is  $m = 20$

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

$$x_1 = \frac{\frac{1}{2}20}{p_1} = \frac{10}{p_1}$$

Demand: how much do I buy at some price  $p_1$ .

**Inverse demand** asks the question, for a particular quantity of a good, what price would be responsible for me buying that amount?

$$x_1 = \frac{10}{p_1}$$

$$p_1 = \frac{10}{x_1}$$

$$p_1 = \frac{10}{1}, p_1 = 10$$

$$p_1 = \frac{10}{2}, p_1 = 5$$

$$(1, 10), (2, 5), (10, 1)$$

$$x_1 = \frac{m}{p_1 + p_2}, \text{ suppose } m = 20, p_2 = 1$$

$$x_1 = \frac{20}{p_1 + 1}$$

$$(p_1 + 1)x_1 = 20$$

$$p_1 + 1 = \frac{20}{x_1}$$

$$p_1 = \frac{20}{x_1} - 1$$

### 1.7.2 Plotting Inverse Demand

Plot the inverse demand, putting the price on the vertical axis.



### 1.7.3 Changes in Other Price

$$x_1 = \frac{m}{p_1 + p_2}$$

In the case of perfect complements as the price of good 2 goes up, demand for good 1 goes down.

$x_1$  is a **complement** for  $x_2$  if when  $p_2$  goes up, the demand for  $x_1$  goes down.

$x_1$  is a **substitute** for  $x_2$  if when  $p_2$  goes up, the demand for  $x_1$  goes up.

$x_1$  is a **neither** for  $x_2$  if when  $p_2$  goes up, the demand for  $x_1$  does not change.

$$x_1 = \frac{m}{p_1 + p_2}$$

$x_1$  is a complement for  $x_2$  since as  $p_2$  goes up, demand for  $x_1$  goes down.

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

$x_1$  is neither a complement nor substitute for  $x_2$  since as  $p_2$  goes up, demand for  $x_1$  does not change.

## 1.8 Quasi-Linear Demand

$$u(x_1, x_2) = \ln(x_1) + x_2$$

Tangency Condition:

$$-\frac{\frac{1}{x_1}}{1} = -\frac{p_1}{p_2}$$

Budget constraint:

$$p_1x_1 + p_2x_2 = m$$

From tangency condition:

$$-\frac{1}{x_1} = -\frac{p_1}{p_2}$$

$$x_1 = \frac{p_2}{p_1}$$

The marshallian demand for  $x_1$ .

It is ordinary since  $p_1$  goes up demand goes down.

It is a substitute for  $x_2$  since as  $p_2$  goes up, demand goes up.