1 Marshallian Demand

$$u\left(x_1, x_2\right) = ln\left(x_1\right) + x_2$$

at the optimum:

This has to be true:

$$MRS = -\frac{p_1}{p_2}$$

By definition the MRS is the negative of the ratio of partial derviative:

$$MRS = -\frac{\frac{\partial(ln(x_1)+x_2)}{\partial x_1}}{\frac{\partial(ln(x_1)+x_2)}{\partial x_2}} = -\frac{\frac{1}{x_1}}{1}$$

$$MRS = -\frac{\frac{1}{x_1}}{1} = -\frac{1}{x_1}$$

Write down the tangency condition:

$$-\frac{1}{x_1} = -\frac{p_1}{p_2}$$

This has to be true (budget condition):

$$p_1 x_1 + p_2 x_2 = m$$

Take the tangecy condition and isolate one of the goods:

$$-\frac{1}{x_1} = -\frac{p_1}{p_2}$$

Get rid of the negtives:

$$\frac{1}{x_1} = \frac{p_1}{p_2}$$

$$x_1 \frac{1}{x_1} = x_1 \frac{p_1}{p_2}$$

$$1 = x_1 \frac{p_1}{p_2}$$

Divide both sides by $\frac{p_1}{p_2}$

$$\frac{p_2}{p_1} = x_1$$

$$x_1 = \frac{p_2}{p_1}$$

To get the demand for the other good, plug it into the budget constraint:

$$p_1 \left(\frac{p_2}{p_1} \right) + p_2 x_2 = m$$

$$p_2 + p_2 x_2 = m$$

$$p_2 \left(1 + x_2 \right) = m$$

$$\left(1 + x_2 \right) = \frac{m}{p_2}$$

$$x_2 = \frac{m}{p_2} - 1$$

$$\left(\frac{p_2}{p_1}, \frac{m}{p_2} - 1 \right)$$

2 Ways to Solve These

Perfect Substitutes

(Budget Condition): $p_1 x_1 + p_2 x_2 = m$

(Endpoint Condition): The optimal bundle will be one of $\left(\frac{m}{p_1}, 0 \right)$, $\left(0, \frac{m}{p_2} \right)$. Which gives more utility?

Perfect Complements $min\left\{\frac{1}{2}x_1, x_2\right\}$, $min\left\{x_1, x_2\right\}$**....**

$(BudgetCondition): p_1x_1 + p_2x_2 = m$

(No-Waste): (Set the things inside the min equal) $\frac{1}{2}x_1 = x_2$ or $x_1 = x_2$...

Everything Else

(Budget Condition): $p_1x_1 + p_2x_2 = m$

(Tangency Condition): $MRS = -\frac{p_1}{p_2}$

$$-\frac{mu_1}{mu_2} = -\frac{p_1}{p_2}$$

$$-\frac{\frac{\partial(u(x_1,x_2))}{\partial x_1}}{\frac{\partial(u(x_1,x_2))}{\partial x_2}} = -\frac{p_1}{p_2}$$

3 Partial Derivatives

$2x_1 + 3x_2$

$$\frac{\partial\left(2x_1 + 3x_2\right)}{\partial x_1} = 2$$

$2x_1x_2$

$$\frac{\partial\left(2x_1x_2\right)}{\partial x_1} = 2x_2$$

$2x_1^2x_2^2$

$$\frac{\partial\left(2x_1^2x_2^2\right)}{\partial x_1} = 2x_2^2\left(2x_1\right) = 4x_2^2x_1$$

4 Budgets

When plotting a budget line, put down the **intercepts** first.

How much x_1 can you afford if you only buy good 1.

How much x_2 can you afford if you only buy good 2.

$$\left(\frac{m}{p_1}, 0\right), \left(0, \frac{m}{p_2}\right)$$

Slope:

$$-\frac{p_1}{p_2}$$

Interpret slope, how much x_2 you give up to get one more x_1.

5 Preferences

$a \succsim a, b \succsim b, c \succsim c, a \succsim b, a \succsim c, b \succsim a$

Complete, Intranstitive

$$b \succsim a, a \succsim c$$

We don't have

$$b \succsim c$$

Here is a complete and transitive one:

$a \succsim a, b \succsim b, c \succsim c, a \succsim b, a \succsim c, b \succsim a, b \succsim c$

$\succ$

$b \succ c, a \succ c$

$\sim$

$a \sim a, b \sim b, c \sim c, a \sim b$

Chain-notation

$a \sim b \succ c$

Best elements: at least as good as anything else in the set.

What is best from $\{a, c\}$. a

What is best from $\{a, b, c\}$. a, b

6 Well-Behaved Prefrences

Monotonicity:

Could these preferences be monotonic?

$$(1, 2) \succ (2, 2)$$

$$(2,2) \sim (3,3)$$

No, both violate the definition.

Convexity:

If you take two bundles you are indifferent between, anything between them (convex combination) has to be at least as good.

$$(3,1) \sim (1,3)$$

What has to be true about $(2,2)$ if these are convex?

$$(2,2) \succsim (3,1)$$

$$(2,2) \succsim (1,3)$$

The same is true about $(1,5,2.5)$ which is also a convex cobination of the originals.

6.1 What this implies about indifference curves

Montonicity: Never slope upwards, preference increases to the north-east

Convexity: If you take a indifference curve and draw a straight line between any two points on it, that line is never below the indiffernce curve. (Bow outwards).

7 Chapter 8 Slutsky decomposition

$$x_1 = \frac{\frac{1}{2}m}{p_1}, x_2 = \frac{\frac{1}{2}m}{p_2}$$

$p_1 = 1, p_2 = 1, m = 20$

Initial optimal bundle:

$$(10,10)$$

Suppose the price of $p_1 = 2$

$$(5,10)$$

Total change in demand for $x_1$is -5.

How do we decompose this into substitution and income effect.

How much income would I need to buy my original bundle at the new prices? Calculate cost of bundle $(10, 10)$ at new prices

$$2 * 10 + 1 * 10 = 30$$

What would they actually buy with this income at the new prices?

$$(7.5, 15)$$

From 10 to 7.5 a decrease of 2.5 could not be due to the income effect. It must be the substitution effect. -2.5

The remaining -2.5 is the income effect.