1 Slutsky Decomposition

Giffen: Has to be inferior. When price goes up, demand goes up.

1.1 Two Effects

There are two effect when a price changes.

Substitution Effect. When the price increases, I will substitute into buying other goods.

Income Effect. When the price increases, effective income goes down. The will lead to a change in demand based on whether the good is normal, or inferior.

1.2 Law of Demand

The substitution effect is always negative. When the price of a good goes up, demand for it will go down **due to substitution**.

While the subtitution effect is always negative after a price increase, the income effect can be negtive (normal) or positive (inferior).

1.3 Three Possibilities

If the price goes up:

Ordinary goods / Normal

Substitution: $-$, Income: $-$

Ordinary goods / Inferior

Substitution: $-$, Income: $+$

This happens when the income effect does not *overwhelm* them substitution effect.

Giffen goods / Inferior

Substitution: $-$, Income: $-$

This happens when the income effect *overwhelms* them substitution effect.

1.4 Slutsky Decomposition - Intuition

How would we measure the substitution effect?

How can hold the income effect constant?

Bundle before price change: $(10, 10)$. Bundle after is $(2, 20)$

Price of good 1 goes up. Imagine after a price change I give you extra money so you can still afford the bundle you were buying before.

Suppose under the new prices and with the extra money you buy $(5, 15)$

The decrease from 10 to 5 can't be due to the income effect.

-5 has to be due to substitution.

-3 has to be due to income.

1.5 Slutsky Decomposition - In Action

$u(x_1, x_2) = x_1 x_2$

$x_1^* = \frac{\frac{1}{2}m}{p_1}, x_2^* = \frac{\frac{1}{2}m}{p_2}$

$p_1 = 1, p_2 = 2, m = 60$

The original bundle purchased is $(30, 15)$

Suppose p_1 increases to $p_1 = 2$.

New demand is: $(15, 15)$

Total Effect: -15. What part of this is due to the substitution effect?

How much money would I need to give this consumer to buy the old bundle at the new prices.

The old bundle $(30, 15)$ costs \$90 under the new prices.

$p_1 = 2, p_2 = 2, \tilde{m} = 90$

The bundle $(30, 15)$ is affordable under the new prices with $\tilde{m} = 90$. **Compensated income.**

What would the consumer buy with this budget?

$x_1^* = \frac{\frac{1}{2}m}{p_1}, x_2^* = \frac{\frac{1}{2}m}{p_2}$

$$\left(\frac{\frac{1}{2}90}{2}, \frac{\frac{1}{2}90}{2}\right) = (22.5, 22.5)$$

Old bundle: $(30, 15)$. Bundle with compensated income under the new prices $(22.5, 22.5)$.

The difference here has to be due to the substitution effect.

$22.5 - 30 = -7.5$ is due to the substitution effect.

$TE = SE + IE$

$-15 = -7.5 + IE$

$IE = -7.5$

The rest, -7.5 is due to the **income effect**.

1.6 Slutsky Decomposition - In Action

$min\{x_1, x_2\}$

$x_1^* = \frac{m}{p_1+p_2}, x_2^* = \frac{m}{p_1+p_2}$

$p_1 = 1, p_2 = 2, m = 60.$

The price of p_1 changes to $p_1 = 2$

a) What bundle does the consumer buy before the price change. **Old Bundle.**

$$\left(\frac{60}{3}, \frac{60}{3}\right) = (20, 20)$$

b) What bundle does the consumer buy after the price change? p_1 changes to $p_1 = 2$

$$\left(\frac{60}{2+2}, \frac{60}{2+2}\right) = (15, 15)$$

Total effect: -5 (new bundle - old bundle).

How much money do we need to give the consumer to buy the old bundle at the new prices. Price of the old bundle:

$$(20, 20)$$

$$2 * 20 + 2 * 20 = 80$$

$\tilde{m} = 80.$

$p_1 = 2, p_2 = 2, \tilde{m} = 80$ what does the consumer buy with this budget?

$$\left(\frac{80}{2+2}, \frac{80}{2+2}\right) = (20, 20)$$

Zero substitution effect. (compensated demand - original demand)

$TE = SE + IE$

$-5 = 0 + IE$

$IE = -5$

The income effect is the entirety of the change in demand -5.

1.7 Quasi-Liner

$p_1 = 1, p_2 = 20, m = 100$

$ln\left(x_1\right) + x_2$

Tangency:k

$$mrs = -\frac{p_1}{p_2}$$

$$-\frac{\frac{1}{x_1}}{1} = -\frac{p_1}{p_2}$$

$$\frac{1}{x_1} = \frac{p_1}{p_2}$$

$$x_1 = \frac{p_2}{p_1}$$

Budget:

$$p_1 x_1 + p_2 x_2 = m$$

Plug tangency into budget:

$$p_1\left(\frac{p_2}{p_1}\right) + p_2 x_2 = m$$

$$x_2 = \frac{m - p_2}{p_2}$$

$x_1 = \frac{p_2}{p_1}, x_2 = \frac{m-p_2}{p_2}$

$p_1 = 1, p_2 = 20, m = 100$

Suppose p_1 increases to 2.

Old Bundle: $\left(\frac{20}{1}, \frac{100-20}{20}\right) = (20, 4)$

New Bundle: $\left(\frac{20}{2}, \frac{100-20}{20}\right) = (10, 4)$

Total Effect -10.

Substitution effect. $\tilde{m}$ (cost of the old bundle at the new prices)

$$\tilde{m} = 2 * 20 + 20 * 4 = 120$$

What do the actually buy at $p_1 = 2, p_2 = 20, \tilde{m} = 120$?

$$x_1 = \frac{20}{2}, x_2 = \frac{120 - 20}{20}$$

$$(10, 5)$$

$SE = 10 - 20 = -10$

$TE = -10$

$SE = -10$

$IE = 0$