1 Cobb Douglass Demand

$x_1^\alpha x_2^\beta$

$$x_1 = \frac{\frac{\alpha}{\alpha+\beta}m}{p_1}, x_2 = \frac{\frac{\beta}{\alpha+\beta}m}{p_2}$$

$$x_1^3 x_2^1$$

$$x_1 = \frac{\frac{3}{4}m}{p_1}, x_2 = \frac{\frac{1}{4}m}{p_2}$$

2 One more Slutsky Decomposition

$x_1^3 x_2^1$

$$x_1 = \frac{\frac{3}{4}m}{p_1}, x_2 = \frac{\frac{1}{4}m}{p_2}$$

$m = 120$, $p_1 = 1, p_2 = 1$

p_2 increases to 2.

Original bundle (amount bought before the price change):

$$\left(\frac{\frac{3}{4}m}{p_1}, \frac{\frac{1}{4}m}{p_2}\right) = \left(\frac{\frac{3}{4}120}{1}, \frac{\frac{1}{4}120}{1}\right) = (90, 30)$$

New bundle (after price change):

$$\left(\frac{\frac{3}{4}m}{p_1}, \frac{\frac{1}{4}m}{p_2}\right) = \left(\frac{\frac{3}{4}120}{1}, \frac{\frac{1}{4}120}{2}\right) = (90, 15)$$

Total effect on good 2 is a 15 unit decrease in demand.

How much of this effect is due to the substitution effect, and how much is due to the income effect?

Suppose at the new prices, I gave you enough income to afford your original bundle.

However this demand differs from the original, cannot due to the income effect. (Income effect has been countacted by the extra income.)

How much income does the consumer need to buy the original bundle at the new prices?

$$\tilde{m} = 1 * 90 + 2 * 30 = 150$$

What does the consumer buy at $p_1 = 1, p_2 = 2, \tilde{m} = 150$.

$$\left(\frac{\frac{3}{4}\tilde{m}}{p_1}, \frac{\frac{1}{4}\tilde{m}}{p_2}\right) = \left(\frac{\frac{3}{4}150}{1}, \frac{\frac{1}{4}150}{2}\right) = (112.5, 18.75)$$

Original: $(90, 30)$, Thought Experiment: $(112.5, 18.75)$

Difference between these, can't be due to income.

$$18.75 - 30 \quad = \quad -11.25$$

-11.25 is due to substitution. The rest, -3.75 is due to the income effect.

3 Buying and Selling

Goal is to move from *exogenous* income to *endogenous*.

Suppose that an apple farmer that has 10 apples, but consumes pies.

3.1 Income to Endowments

w_1 is the endowment of good 1

w_2 is the endowment of good 2

$w_1 = 20$, $w_2 = 0$ apple farmer starts with 20 apples.

This is really like a "starting" bundle $(20, 0)$.

p_1, p_2 are prices as before.

Budget line with exogenous income:

$$p_1 x_1 + p_2 x_2 = m$$

Value of the consumers endowment:

$$p_1 w_1 + p_2 w_2$$

For the farmer with 20 apples:

$$p_1(20) + p_2(0) = 20p_1$$

The cost of the bundle they buy has to be less than or equal to the value of their endowment:

$$p_1x_1 + p_2x_2 \leq p_1w_1 + p_2w_2$$

The budget line:

$$p_1x_1 + p_2x_2 = p_1w_1 + p_2w_2$$

Suppose we have $w_1 = 20$, $w_2 = 0$ and $p_1 = 1, p_2 = 1$

Their budget equation:

$$x_1 + x_2 = 1(20) + 1(0)$$

$$x_1 + x_2 = 20$$

Price of p_1 increases to 2.

$$2x_1 + x_2 = 40$$

The income (value of the endowment) changes as the prices change.

3.2 Graphing Budget

Slope of the budget is still $-\frac{p_1}{p_2}$.

How much x_1 can I have if I only buy x_1

$$p_1x_1 + p_2x_2 = p_1w_1 + p_2w_2$$

$$p_1x_1 + p_2(0) = p_1w_1 + p_2w_2$$

x_1 intercept

$$x_1 = w_1 + \frac{p_2w_2}{p_1}$$

x_2 intercept "How much x_2 can I have if I only buy x_2"

$$x_2 = w_2 + \frac{p_1w_1}{p_2}$$

3.3 Gross/Net Demand

3.4 Net Buyer/Seller

A consumer is a *net* buyer of x_i if $x_i > w_i$.

A consumer is a *net* seller of x_i if $x_i < w_i$.

If you are a net buyer of one good, you are a net seller of the other.

3.5 Gross Demand and Net Demand

x_1, x_2 **gross demand** amount you want to end up with

$(x_1 - w_1)$ **net demand** how much extra do I need to get the amount I want.

Suppose $(w_1, w_2) = (5, 5)$

Gross demand: $(x_1, x_2) = (10, 0)$

Net demand for $x_1 = 10 - 5 = 5$

Net demand for $x_2 = 0 - 5 = -5$

If net demand is positive, the consumer is a buyer.

If net demand is negative, the consumer is a seller.

3.5.1 Changing Prices

Unlike with an endowment of money, when a price changes, the budget line "pivots" through the endowment point.

3.5.2 Changing Prices and Net Buyers/Sellers

If a consumer is a seller of a good, and the price of that good goes up, they **remain a net seller** and a **strictly better off**.

If a consumer is a buyer of a good, and the price of that good goes down, they **remain a net buyer** and a **strictly better off**.

3.5.3 Example

$min\left\{\frac{1}{2}x_1, x_2\right\}$ $p_1 = 1, p_2 = 1, w_1 = 40, w_2 = 0$

Write the consumer's budget equation:

$$p_1x_1 + p_2x_2 = p_1w_1 + p_2w_2$$

$$x_1 + x_2 = 40$$

What is the optimal bundle?

$min\left\{\frac{1}{2}x_1, x_2\right\}$

No waste condition:

$$\frac{1}{2}x_1 = x_2$$

Budget equation:

$$x_1 + x_2 = 40$$

Solve these:

$$x_1 + \frac{1}{2}x_1 = 40$$

$$\frac{3}{2}x_1 = 40$$

$$x_1 = \frac{80}{3}$$

Plug this back into the no waste condition:

$$\frac{1}{2}\frac{80}{3} = x_2$$

$$\frac{80}{6} = x_2$$

Optimal bundle (gross demand):

$$(26.6667, 13.3333)$$

What is the net demand:

$$(26.6667 - 40, 13.3333 - 0)$$

$$\left(-13\frac{1}{3}, 13\frac{1}{3}\right)$$

Net seller of x_1 and a net buyer of x_2.

Suppose the price of x_1 increases to $p_1 = 2$. Is the consumer now a net seller of a net buyer of x_1?

Are they better off?

Yes, they remain a net seller of x_1 *and they are strictly better off.*