1 Cobb Douglass Demand

$x_1^\alpha x_2^\beta$

$$\frac{\partial\left(x_1^\alpha x_2^\beta\right)}{\partial x_1} = \alpha x_1^{\alpha-1} x_2^\beta = \alpha x_1^\alpha x_2^\beta x_1^{-1}$$

$$\frac{\partial\left(x_1^\alpha x_2^\beta\right)}{\partial x_2} = \beta x_1^\alpha x_2^{\beta-1} = \beta x_1^\alpha x_2^\beta x_2^{-1}$$

$$MRS = -\frac{\alpha x_1^\alpha x_2^\beta x_1^{-1}}{\beta x_1^\alpha x_2^\beta x_2^{-1}} = -\frac{\alpha}{\beta}\frac{x_1^{-1}}{x_2^{-1}} = -\frac{\alpha}{\beta}\frac{x_2}{x_1}$$

$$MRS = -\frac{\alpha}{\beta}\frac{x_2}{x_1}$$

$$-\frac{\alpha}{\beta}\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

$$p_1 x_1 + p_2 x_2 = m$$

Solving these together:

$$x_1 = \frac{\frac{\alpha}{\alpha+\beta} m}{p_1}, x_2 = \frac{\frac{\beta}{\alpha+\beta} m}{p_2}$$

2 One more Slutsky Decomposition

$x_1^3 x_2^1$

$$x_1 = \frac{\frac{3}{4} m}{p_1}, x_2 = \frac{\frac{1}{4} m}{p_2}$$

$m = 480$, $p_1 = 1, p_2 = 1$

p_2 increases to $p_2 = 2$.

Find the original bundle:

$$x_1 = \frac{\frac{3}{4}480}{1} = 360, x_2 = \frac{\frac{1}{4}480}{1} = 120$$

$$(360, 120)$$

Bundle after the price change:

$$x_1 = \frac{\frac{3}{4}480}{1} = 360, x_2 = \frac{\frac{1}{4}480}{2} = 60$$

$$(360, 60)$$

Total effect of the price change is a 60 unit decrease in demand for good 2.

The substitution effect is measured by the consumers change in demand from the original bundle under the new price but if they had **enough income** to buy original bundle.

The cost of the original bundle under the new prices:

$$\tilde{m} = 360 * 1 + 120 * 2 = 600$$

$p_1 = 1, p_2 = 2, \tilde{m} = 600$

$$x_1 = \frac{\frac{3}{4}600}{1} = \frac{1800}{4} = 450, x_2 = \frac{\frac{1}{4}600}{2} = \frac{600}{8} = 75$$

$$(450, 75)$$

Originally they bought 120 x_2 and they buy 75 under this thought experiment.

$$75 - 120 = -45$$

Since the total effect if a decrease of 60 and the substitution effect is a decrease of 45, the income effect must be a decrease of 15.

3 Buying and Selling

Up until now, income is *exogenous*. “Determined outside of the model”.

We want to *endogenize* income. “Make it part of the model. Have income depend on prices.”

3.1 Income to Endowments

(income) $m \to (w_1, w_2)$ (endowments)

An endowment is an amount of x_1 and x_2 that the consumer *starts with.*

x_1 apples, x_2 crusts.

$(w_1, w_2) = (20, 0)$ says "the consumer starts with 20 apples"

$(w_1, w_2) = (0, 10)$ says "the consumer starts with 10 crsuts"

The endowments are the "stuff" the consumer brings to model to sell in order to earn their income.

$(w_1, w_2) = (5, 10)$ the consumer starts with 5 x_1 and 10 x_2.

3.2 The budget equation

$(20, 0)$ how much is this endowment worth?

$p_1 = 1, p_2 = 1$

$$20 * p_1 + 0 * p_2 = 20p_1 = 20$$

$p_1 = 2, p_2 = 1$

$$20 * p_1 + 0 * p_2 = 20p_1 = 40$$

What can the consumer afford?

$$p_1x_1 + p_2x_2 \leq m$$

Replace m with the value of the consumer's endowment:

$$p_1x_1 + p_2x_2 \leq p_1w_1 + p_2w_2$$

Budget Line:

$$p_1x_1 + p_2x_2 = p_1w_1 + p_2w_2$$

In the case of $p_1 = 1, p_2 = 1, w_1 = 20, w_2 = 0$

$$x_1 + x_2 = 20$$

Suppose p_1 changes to 2.

$$2x_1 + x_2 = 40$$

3.3 Graphing Budget

The budget line will always include (w_1, w_2).

Slope of the budget line is $-\frac{p_1}{p_2}$.

When a price changes, the slope of the budget line will change, but the line will pivot through the point (w_1, w_2).

3.4 Net Buyer/Seller

If $x_i > w_i$ **net buyer** of good i

If $x_i < w_i$ **net seller** of good i

3.5 Gross/Net Demand

(x_1, x_2) "bundle you want" **gross demand**

$(x_1 - w_1, x_2 - w_2)$ **net demand.** Difference between gross demand and endowment.

$(w_1, w_2) = (5, 5)$, $(x_1, x_2) = (10, 0)$

Gross demand is $x_1 = 10$, $x_2 = 0$

Net demand is $x_1 - w_1 = 10 - 5 = 5$, $x_2 - w_2 = 0 - 5 = -5$

When net demand is positive, the consumer is a **buyer** of that good.

When net demand is negative, the consumer is a **seller** of that good.

If you are a buyer of one good, you have to be a seller of the other.

3.5.1 Changing Prices and Net Buyers/Sellers

If a consumer is a **seller** of a good, and the price of that good **goes up**:

1. The consumer **remains a net seller.**

2. They are **strictly better off**.

If a consumer is a **buyer** of a good, and the price of that good **goes down**:

1. The consumer **remains a net buyer.**

2. They are **strictly better off**.

3.5.2 Example

$min\left\{\frac{1}{2}x_1, x_2\right\}$ $p_1 = 1, p_2 = 1, w_1 = 120, w_2 = 0$