# 1 Intertemporal Choice

Model to study how people chosen when to spend money? Borrowing/Saving.

## 1.1 Bundles

$(c_1, c_2)$

$c_1$ dollars of consumption in period 1

$c_2$ dollars of consumption in period 2

*Budget* Set? Based on a endowment of money today and money in the future.

**Income Stream**

$(m_1, m_2)$

$m_1$ income in period 1

$m_2$ income in period 2

$(m_1, m_2) = (100000, 0)$

$(m_1, m_2) = (100,000, 1,000,000)$

## 1.2 Budget No Interest

All they can do is borrow money from their future earnings (at zero interest) and save money for the futrure.

Suppose their income stream is $(m_1, m_2) = (100000, 0)$

Save 50000 for the future. $(c_1, c_2) = (50000, 50000)$

Suppose income stream is

$(m_1, m_2) = (100,000, 1,000,000)$

Borrow $400,000$ of future earnings

$(c_1, c_2) = (100000 + 400000, 1000000 - 400000)$

$(c_1, c_2) = (500000, 600000)$

$$c_1 + c_2 \leq m_1 + m_2$$

Budget line:

$$c_1 + c_2 = m_1 + m_2$$

Slope: $-1$

$u(c_1, c_2) = c_1 c_2$ "consumption smooth"

$(m_1, m_2) = (100000, 0)$

$$c_1 + c_2 = 100000$$

Tangency Condition:

$$-\frac{c_2}{c_1} = -1$$

$$c_1 = c_2$$

Budget Constraint:

$$c_1 + c_2 = 100000$$

Solve these together, plugging in the trangency into budget:

$$c_1 + (c_1) = 100000$$

$$c_1 = 50000$$

Optimal bundle $(c_1, c_2) = (50000, 50000)$.

### 1.3 Budget

Normally, when you save money, you get back more than saved.

Normally, when you borrow money, you pay back more than you borrow.

Interest rate on borrowing and savings is the same.

$r$ the extra amount you get back in period 2 for every dollar you save

$r$ the extra amount you pay back in period 2 for every dollar you borrow

Suppose the consumer is going to save money.

$$c_1 < m_1$$

Amount of savings is $m_1 - c_1$

The amount they consume tomorrow is:

$$c_2 = m_2 + (1+r)(m_1 - c_1)$$

Suppose $r = \frac{1}{2}$

$m_1 = 1000, m_2 = 1000$

$c_1 = 500$

$$c_2 = 1000 + (1000 - 500) + r(1000 - 500)$$

$$c_2 = 1000 + 500 + \frac{1}{2}(500) = 1750$$

$$(500, 1750)$$

$$c_2 = m_2 + (1+r)(m_1 - c_1)$$

$$c_2 = m_2 + (1+r)m_1 - (1+r)c_1$$

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

Assume the consumer borrows money.

The amount you take out in a loan is $(c_1 - m_1)$

Tomorrow you have to pay back $(c_1 - m_1)$ plus $r(c_1 - m_1)$. $(1+r)(c_1 - m_1)$

$$c_2 = m_2 - (1+r)(c_1 - m_1)$$

$$c_2 = m_2 - (1+r)c_1 + (1+r)m_1$$

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

## 1.4 Plotting the Budget

The budget constaint is a line with slope of $-(1+r)$ that passes through the endowment $(m_1, m_2)$.

Endpoints:

$c_2$ endpoint is how much the stream of income can afford the consumer in terms of period 2 consumption.

$m_2 + (1+r)\, m_1$ **future value of income**

$c_1$ endpoint is how much the stream of income can afford the consumer in terms of period 1 consumption.

$m_1 + \frac{m_2}{1+r}$ **present value of income**

Notice, if you take out a loan of $\frac{m_2}{1+r}$, in period 2 you owe $(1+r)\frac{m_2}{1+r} = m_2$.This is the biggest loan you can take out.

## 1.5 Interest Rate Changes

$(1+r)$ is really acting as our "price" of good 1.

An interest rate increase is "like" an increase in $p_1$ in our previous model.

$r$ goes up, the slope of the budget becomes steeper, you have to give up more $c_2$ to get an extra unit of $c_1$.

## 1.6 Interest Rate Changes and Borrowers /Savers

Borrow is someone for whom $c_1 > m_1$

For a borrower if the interest rate goes down.

1. They will remain a borrower.

2. They will be strictly better off.

Saver is someone for whom $c_1 < m_1$

For a saver if the interest rate goes up.

1. They will remain a saver.

2. They will be strictly better off.

### 1.6.1 Example Problem

$u(c_1, c_2) = c_1 c_2$. $m_1 = 100000, m_2 = 0, r = \frac{1}{2}$

Tangency:

$$-\frac{c_2}{c_1} = -1.5$$

$$\frac{c_2}{c_1} = 1.5$$

$$c_2 = 1.5c_1$$

Budget:

$$1.5c_1 + c_2 = 150000$$

Solve these:

$$1.5c_1 + (1.5c_1) = 150000$$

$$3c_1 = 150000$$

$$c_1 = \frac{150000}{3} = 50000$$

$$c_2 = 1.5\,(50000) = 75000$$

$$(50000, 75000)$$

Suppose the interest rate increases to $r = \frac{3}{4}$. Will the consumer be a borrow or save? Are they better off?

They remain a saver, and they will be strictly better off.

Try redoing this problem with $r = \frac{3}{4}$.