1 Example from Last Class

$min\left\{\frac{1}{2}x_1, x_2\right\}$ $p_1 = 1, p_2 = 1, w_1 = 120, w_2 = 0$

m

(w_1, w_2)

$p_1x_1 + p_2x_2 = p_1w_1 + p_2w_2$

a) What is the consumer's budget equation?

$x_1 + x_2 = 120$

a.2) Along the budget line, how much x_2 does the consumer neet to give up to get one more unit of x_1?

-1

$-\frac{p_1}{p_2}$

b) What is the consumer's gross demand? (What bundle is optimal for them)?

No waste condition: $\frac{1}{2}x_1 = x_2$

Budget constraint: $x_1 + x_2 = 120$

Solve these:

$x_1 + \left(\frac{1}{2}x_1\right) = 120$

$$x_1 + \left(\frac{1}{2}x_1\right) = 120$$

$$\frac{3}{2}x_1 = 120$$

$$x_1 = 80$$

Plug this back into the no-waste condition:

$$\frac{1}{2}(80) = x_2$$

$$x_2 = 40$$

Gross demand:

$$(80, 40)$$

c) What is the net demand?

$$x_1 - w_1 = 80 - 120 = -40$$

$$x_2 - w_2 = 40 - 0 = 40$$

d) Are a net buyer or net seller x_1?

Net seller of x_1. Net buyer of x_2.

e) Suppose the price of good 1 goes up to $p_1 = 2$. Is this consumer a net buyer or a net seller? Are they better off?

Remain a net seller and they will be better off.

2 Intertemporal Choice

Model to study how people chosen when to spend money? Borrowing/Saving.

2.1 Bundles

(c_1, c_2)

c_1 amount of money spent in consumption in period 1.

c_2 amount of money spent in consumption in period 2.

(m_1, m_2) steam of income

m_1 is income in period 1

m_2 is income in period 2

2.2 Budget No Interest

$m_1 = 1000, m_2 = 1000$

$c_1 + c_2 = m_1 + m_2$

$u(c_1, c_2) = c_1 c_2$

$$-\frac{c_2}{c_1} = -1$$

$$c_1 = c_2$$

(Consumption Smoothing)

Budget:

$$c_1 + c_2 = 2000$$

$$c_1 + (c_1) = 2000$$

$$c_1 = 1000, c_2 = 1000$$

2.3 Budget

Borrowing isn't free, and savings gives you back more than you save.

r interest rate.

Borrow a dollar, you pay back $\$\,(1+r)$ in the next period.

If $r = 0.05$ if you borrow 1000 then next period you owe 1050.

Save a dollar, you get back $\$\,(1+r)$ in the next period.

If $r = 0.05$ if you save 1000 then next period you get 1050.

2.4 Constructing the budget constraint.

Suppose a consumer saves money.

$c_1 < m_1$ they have saved $m_1 - c_1$

How much do they get to consume in period 2?

m_2 income in period 2. $(1+r)(m_1 - c_1)$ is return on savings

$$c_2 = m_2 + (1+r)(m_1 - c_1)$$

$$c_2 = m_2 + (1+r)\,m_1 - (1+r)\,c_1$$

$$(1+r)\,c_1 + c_2 = (1+r)\,m_1 + m_2$$

$p_1 = (1+r), p_2 = 1 \; -\frac{p_1}{p_2} = -(1+r)$

Suppose a consumer borrows money.

$c_1 > m_1$ they have borrow $c_1 - m_1$

How much do they get to consume in period 2?

$$c_2 = m_2 - (1 + r)(c_1 - m_1)$$

$$c_2 = m_2 - (1 + r)c_1 + (1 + r)m_1$$

$$(1 + r)c_1 + c_2 = (1 + r)m_1 + m_2$$

2.5 Plotting the Budget

The budget line is a line through the endowment with slope $-(1 + r)$.

Intercepts:

c_1 intercept. How much c_1 can I have if I only consume in period 1?

$$(1 + r)c_1 + c_2 = (1 + r)m_1 + m_2$$

$$(1 + r)c_1 + 0 = (1 + r)m_1 + m_2$$

$$c_1 = m_1 + \frac{m_2}{(1 + r)}$$

Present value of income.

c_2 intercept. How much c_2 can I have if I only consume in period 2?

$$(1 + r)c_1 + c_2 = (1 + r)m_1 + m_2$$

$$c_2 = (1 + r)m_1 + m_2$$

Future value of income.

2.6 Interest Rate Changes

If r increases, the budget line pivots through the endowmnet but becomes steeper (it costs more c_2 to get extra c_1)

If r decreases, the budget line pivots through the endowmnet but becomes shallower (it costs less c_2 to get extra c_1)

2.7 Interest Rate Changes and Borrowers /Savers

The area to the right of the endowment on the budget line is where the consumer is a **borrower, $c_1 > m_1$.**

If you are a borrower and the interest rate decreases. You will reamin a borrower and be strictly better off.

The area to the left of the endowment on the budget line is where the consumer is a **saver, $c_1 < m_1$.**

If you are a saver and the interest rate increases. You will reamin a saver and be strictly better off.

2.7.1 Example Problem

$u(c_1, c_2) = c_1c_2$. $m_1 = 100000, m_2 = 0, r = \frac{1}{2}$

a) Write the consumer budget equation/line?

$$1.5c_1 + c_2 = 1.5\,(100000) + 0$$

$$1.5c_1 + c_2 = 150000$$

b) If the consumer wants 1 more dollar of c_1 how much c_2 do they have to give up along the budget line?

$$1.5$$

$$-(1 + r) = -1.5$$

c) What is the optimal bundle of c_1, c_2?

c_1c_2

Tangency:

$$-\frac{c_2}{c_1} = -1.5$$

$$c_2 = 1.5c_1$$

Budget:

$$1.5c_1 + (1.5c_1) = 150000$$

$$3c_1 = 150000$$

$$c_1 = 50000$$

To get c_2, plug this back into the tangency condition:

$$c_2 = 1.5c_1$$

$$c_2 = 1.5\,(50000) = 75000$$

$$(50000, 75000)$$