1 Preferences

1.1 Bads

Any time you prefere less of something, we call it a "bad".

Suppose x_1 was a bad and x_2 is a "good". We might have:

$$(1,2) \succ (2,2)$$

When both things are "good" then indifference slope downwards.

When one thing is a bad, and the other is a good, then indifferences slope upwards.

When both things are bads, the indifferences slope downward, but preferences increases as you move towards the origin.

1.2 Indifference Curves Cannot Cross

What restrictions does complteness and transitivity put on the possible indifference curves?

If preferences are complete and transitive, Two distinct indifferences cannot cross.

2 Utility

$$(2,1) \succsim (1,1)$$

Normally we want to represent preferences in a way that makes them easy to work with.

Remy doesn't care about flavor of ice cream he only cares about the total amount.

$$(2,1) \succ (1,1), (0,1) \succ (0,0), (1,0) \sim (0,1)$$

This works sometimes, but we a better way to describe preferences without ambiguity.

To do that we use a utility function.

2.1 Definition

A utility function is a way of **representing preferences** over bundles.

It a function $u(x_1, x_2)$ such that if

$$(x_1, x_2) \succsim \left(x_1^{'}, x_2^{'}\right)$$

Then

$$u(x_1, x_2) \geq u\left(x_1^{'}, x_2^{'}\right)$$

And

$$u(x_1, x_2) \geq u\left(x_1^{'}, x_2^{'}\right)$$

We can infer

$$(x_1, x_2) \succsim \left(x_1^{'}, x_2^{'}\right)$$

A utility functyion is a way of scoring bundles such that better bundles get a higher score.

2.2 Example

$a \sim b \succ c \sim d \succ e$

$u(a) = 3, u(b) = 3, u(c) = 2, u(d) = 2, u(e) = 1$

Is it true that any two bundles if one is strictly prefered it gets a higher utility?

If two bundles are indifferent, the get the same utility?

This utility function represents these underlying preferences.

$u(a) = 300, u(b) = 300, u(c) = 50, u(d) = 50, u(e) = 0$

There are many utility functions that represent the same preferences.

Utility Funtions are inherently ordinal. There is no meaning the magnituted, all that matters is the comparisons.

2.3 Perfect Subsitutes

Remy only care about total ice cream but not the flavor.

(x_1, x_2)

$u(x_1, x_2) = x_1 + x_2$

Check this for some bundles:

$(2,1) \succ (1,1)$,$u(2,1) = 3, u(1,1) = 2$. $u(2,1) > u(1,1)$

$(3,1) \sim (1,3)$,$u(3,1) = 4, u(1,3) = 4$

The utility function represents Remy's preferences.

Other functions represent Remy's Preferences as well.

$u(x_1, x_2) = (x_1 + x_2)^2$

$(2,1) \succ (1,1)$,$u(2,1) = (2+1)^2 = 9, u(1,1) = (1+1)^2 = 4$

$(3,1) \sim (1,3)$,$u(3,1) = 16, u(1,3) = 16$

This one still represents the prferences, but it is mathematically more complex.

Perfect Subsitutes Preferences in General. Cononical Utility Representation for **Perfect Substitutes**

$u(x_1, x_2) = ax_1 + bx_2$

Prefect Substitutes Preferences over Dozen Eggs (x_1) and $Eggs$ (x_2).

$$u(x_1, x_2) = 12x_1 + x_2$$

$(1,0) \sim (0,12)$

2.4 Perfect Complements

Someone consumes left (x_1) and right shoes (x_2). They only care about the number of pairs of shoes.

$u(x_1, x_2) = min\{x_1, x_2\}$

$(4,5) \sim (5,4)$ they have the same utility.

$u(4,5) = 4, u(5,4) = 4$

$(3,2) \succ (1,1)$

$u(3,2) = 2, u(1,1) = 1$

Someone consumes only apple pies. An apple pie consists of 2 apples (x_1) and 1 crust (x_2).

$(4,2) \sim (5,2)$

$(4,2) \succ (2,1)$

$u(x_1,x_2) = \left\{\frac{1}{2}x_1, x_2\right\}$

$u(4,2) = min\{2,2\} = 2$

$u(5,2) = min\left\{\frac{1}{2}5, 2\right\} = min\{2.5,2\} = 2$

$u(2,1) = min\{1,1\} = 1$

Perfect Complements Preferences Can represented with this utility:

Consume combinations of goods, where the "useful combination" requires a of x_1 and b of x_2. Then you can represent this with this cononical representation of perfect complements prefereces:

$$u(x_1,x_2) = min\left\{\frac{1}{a}x_1, \frac{1}{b}x_2\right\}$$

2.5 Transformations

If we take a utility function, and transform it by taking an increasing function of that utility function, the result will still represent the underlying preferences.

$$u(x_1,x_2) = 2x_1 + 3x_2$$

$$(u(x_1,x_2))^2 = (2x_1 + 3x_2)^2$$

$$10u(x_1,x_2) = 10(2x_1 + 3x_2)$$

We call these **monotonic transformations.**

$$u(x_1,x_2) = x_1x_2$$

$$ln(u(x_1,x_2)) = ln(x_1x_2) = ln(x_1) + ln(x_2)$$

2.6 MRS From Utility

$u(x_1,x_2) = x_1 + x_2$

Marginal Utility x_1 is the partial derivative of $u(x_1,x_2)$ with respect to x_1.

Marginal Utility x_1 is the partial derivative of $u(x_1,x_2)$ with respect to x_2.

The ratio of Mu_1 and Mu_2 tells us relatively how much the consumer cares about the two goods.

Slope of the indifference curve MRS

$$MRS = -\frac{mu_1}{mu_2} = -\frac{\frac{\partial(u(x_1,x_2))}{\partial x_1}}{\frac{\partial(u(x_1,x_2))}{\partial x_2}}$$

Suppose $mu_1 = 2$ and $mu_2 = 1$

This consumer cares twice as much about good 1.

The should be willing to give up 2 units of good 2 to get 1 unit of good 1 which tells us the slope of hte indifference curve should be -2.

$$MRS = -\frac{2}{1} = -2$$

$u(x_1, x_2) = x_1 + x_2$

$$MRS = -\frac{mu_1}{mu_2} = -\frac{\frac{\partial(x_1+x_2)}{\partial x_1}}{\frac{\partial(x_1+x_2)}{\partial x_2}} = -\frac{1}{1} = -1$$

2.7 Cobb Douglass

The conconical form for cobb douglass preferences:

$$u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$$

Simpler cobb douglass:

$$u(x_1, x_2) = x_1 x_2$$

How does this consumer trade off between the goods?

$$MRS = -\frac{\frac{\partial(x_1 x_2)}{\partial x_1}}{\frac{\partial(x_1 x_2)}{\partial x_2}}$$

$\frac{\partial(x_1 x_2)}{\partial x_1} = x_2, \frac{\partial(x_1 x_2)}{\partial x_2} = x_1$

$$MRS = -\frac{x_2}{x_1}$$