1 Preferences

1.1 Bads

"Bad" refers to something you dislike.

Suppose x_1 is a "good" and x_2 is a "bad", we might have:

$$(1,2) \succ (2,2)$$

When this happens, (one thing is bad and one is good) indifference curves slope upwards.

When both are "good" the indifference curves will slope downwards.

1.2 Indifference Curves Cannot Cross

If preferences are **complete** and **transitive**, then **two distinct indifference curves cannot cross**.

2 Utility

$x \succ y \succ z \sim w$

Remy like more ice cream rather than less but does not care about flavor.

$$(3,0) \succ (0,2)\,, (2,2) \sim (0,4)$$

Remy likes more ice cream, but doesn't want too much of any one flavor.

$$(2,2) \succ (0,4)$$

$$(2,2)?\,(0,10)$$

We need a way of representing preferences.

We use a **utility function** to describe preferences.

A mathematical representation of preferences using numbers.

2.1 Definition

$u(x_1, x_2)$ if a function that assigns a score to every bundle (x_1, x_2) such that

if $(x_1, x_2) \sim \left(x_1^{'}, x_2^{'}\right)$, then:

$$u(x_1, x_2) = u\left(x_1^{'}, x_2^{'}\right)$$

if $(x_1, x_2) \succ \left(x_1^{'}, x_2^{'}\right)$, then:

$$u(x_1, x_2) > u\left(x_1^{'}, x_2^{'}\right)$$

2.2 Example

$a \sim b \succ c \sim d \succ e$

$u(a) = 3, u(b) = 3, u(c) = 2, u(d) = 2, u(e) = 1$

This utility function represents the preferences above. Better bundles get higher scores, indifference bundles get the same score.

$b \succ d$ and $u(b) > u(d)$

$a \sim b \succ c \sim d \succ e$

$u(a) = 3, u(b) = 3, u(c) = 2, u(d) = 4, u(e) = 1$

Doesn't represent the preferences.

$d \sim c$ and yet $u(d) > u(c)$

$b \succ d$ and yet $u(d) > u(b)$

$a \sim b \succ c \sim d \succ e$

$u(a) = 100, u(b) = 100, u(c) = 10, u(d) = 10, u(e) = 0$

Another utility function that represents the preferences.

The difference between utilities of two bundles **does not represent how much more a consumer likes one over another.**

Utility functions are inherently **ordinal**. The numbers only represent relative preference, but the magnitude does not matter.

2.2.1 Perfect Subsitutes

Remy's preference over bowls of ice cream are that he wants more but doesn't care about flavor.

(x_1, x_2)

Let's represent his preferences for a bowl with the total number of scoops.

$u(x_1, x_2) = x_1 + x_2$

$(4, 0) \sim (0, 4)$ and $u(4, 0) = u(0, 4) = 4$

$(2, 1) \succ (1, 1)$ and $u(2, 1) = 3 > u(1, 1) = 2$

Preferences for packs of a dozen eggs x_1 and single eggs x_2 if I only care about total eggs:

$u(x_1, x_2) = 12x_1 + x_2$

Any time we have a utility function like the one below, the preferences it represents are **perfect substitutes** preferences.

$$u(x_1, x_2) = ax_1 + bx_2$$

2.2.2 Perfect Complements

Consumer only consumes left and right shoes. Only cares about the total (usable) pairs of shoes.

(x_1, x_2)

$(2, 1)$ 1 usable pair

$(1, 2)$ 1usable pair

$(2, 2)$ 2 usable pairs

$(10, 4)$ 4 usable pairs

$$u(x_1, x_2) = min\{x_1, x_2\}$$

$u(2, 1) = min\{2, 1\} = 1$

$u(1, 2) = min\{1, 2\} = 1$

A consumer eats only apple pie and an apple pie consists of 2 apples (x_1) and 1 crust (x_2).

$(2, 1)$: 1 pie

$(3, 1)$: 1 pie

$(3, 1.5)$: 1.5 pies

$(1, 1)$: 0.5 pies

$(6, 4)$: 3 pies

$$u(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$$

The indifference curves here are L-shaped with kinks that follow the line $\frac{1}{2}x_1 = x_2$

In general, if I need a units of x_1 per combination, and b units of x_2 per combination then the utility function is:

$$u(x_1, x_2) = min\left\{\frac{1}{a}x_1, \frac{1}{b}x_2\right\}$$

If you need three plums to plum pie and one crust:

$$u(x_1, x_2) = min\left\{\frac{1}{3}x_1, x_2\right\}$$

2.2.3 Transformations

$u(x_1, x_2) = x_1 + x_2$

$u(x_1, x_2) = x_1 + x_2 + 10, \ f(z) = z + 10$

$$(2,2) \succ (2,1)$$

$$u(2,2) = 14, u(2,1) = 13$$

$u(x_1, x_2) = 2(x_1 + x_2), f(z) = 2z$

$$(2,2) \succ (2,1)$$

$$u(2,2) = 8, u(2,1) = 6$$

All of these are monotonic transformations of eachother, they all represent the same preferences.

$$u(x_1, x_2)$$

$$f(u(x_1, x_2))$$

If f is a strictly increasing function, then result represents the same preferences.

$$u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{2}{3}}$$

Transform it by taking the ln:

$$u(x_1, x_2) = \frac{1}{2} ln(x_1) + \frac{2}{3} ln(x_2)$$

2.2.4 MRS From Utility

Mrs stands for Marginal Rate of Stubstitution (slope of an indiffernce curve)

$$u(x_1, x_2) = 2x_1 + x_2$$

If we increase x_1 by one unit, utility goes up by 2.

Marginal utility of x_1. $mu_1 = 2$

$$mu_1 = \frac{\partial (u(x_1, x_2))}{\partial x_1}$$

If we increase x_2 by one unit, utility goes up by 1.

Marginal utility of x_2. $mu_2 = 1$

$$mu_2 = \frac{\partial (u(x_1, x_2))}{\partial x_2}$$

$$-\frac{mu_1}{mu_2} = -2$$

Suppose instead he likes x_2 three times more than x_1.

$$u(x_1, x_2) = x_1 + 3x_2$$

$$mu_1 = 1, mu_2 = 3$$

$$MRS = -\frac{mu_1}{mu_2} = -\frac{1}{3}$$

The slope of an indifference curve at a point, will always be measured by the ratio of marginal utility at that point.