1 constrained optimization

The constained optimal point on a monotonic function cannot be in the interior of the constraint.

The bundle that maximizes their utility subject to their budget constraint, **must be on the budget line**.

an optimal bundle must live on a contour that does not pass into the **interior** of the budget set

In Terms of Economics: **The bundle must be on the budget line and on an indifference curve that just touches but does not cross into the budget set.**

1.1 Perfect Substitutes

$u(x_1, x_2) = 2x_1 + x_2$

$p_1 = 1, p_2 = 1, m = 10$

Slope of the budget line: -1

Slope of the indifference curve: -2

$$-1 = -2$$

$$(10, 0)$$

One of the two "corners" (intercepts of the budget) will always be optimal. Just figure out which one.

$$(10, 0), (0, 5)$$

$$u(10, 0) = 10$$

$$u(0, 5) = 5$$

$u(x_1, x_2) = 3x_1 + 2x_2, p_1 = 4, p_2 = 3, m = 12$

$$(3, 0), (0, 4)$$

$$u(3, 0) = 9, u(0, 4) = 8$$

Optimal:

$$(3, 0)$$

1.2 Perfect Complements

$u(x_1, x_2) = min\{x_1, x_2\}, p_1 = 1, p_2 = 2, m = 12$

$$x_1 = x_2$$

$$x_1 + 2x_2 = 12$$

(x_1, x_2)

$$x_1 = x_2$$

$$x_1 + 2(x_1) = 12$$

$$3x_1 = 12$$

$$x_1 = 4$$

$$4 = x_2$$

$$(4, 4)$$

1.3 Cobb Douglass

$u(x_1, x_2) = x_1 x_2, p_1 = 2, p_2 = 1, m = 12$

Budget condition (bundle must cost all of the consumers income).

$$2x_1 + x_2 = 12$$

Equal slope condition.

$$mrs = -\frac{p_1}{p_2}$$

$$-\frac{\frac{\partial(x_1 x_2)}{\partial x_1}}{\frac{\partial(x_1 x_2)}{\partial x_2}} = -\frac{p_1}{p_2}$$

$\frac{\partial(x_1 x_2)}{\partial x_1} = x_2, \frac{\partial(x_1 x_2)}{\partial x_2} = x_1$

$$-\frac{x_2}{x_1} = -\frac{2}{1}$$

Two equations:

$$2x_1 + x_2 = 12$$

$$-\frac{x_2}{x_1} = -\frac{2}{1}$$

Let's solve for x_2 in the equal slope condition:

$$-\frac{x_2}{x_1} = -\frac{2}{1}$$

$$\frac{x_2}{x_1} = 2$$

$$x_1 \frac{x_2}{x_1} = x_1 2$$

$$x_2 = 2x_1$$

Plug this into the budget constraint:

$$2x_1 + x_2 = 12$$

$$2x_1 + (2x_1) = 12$$

$$4x_1 = 12$$

$$x_1 = 3$$

Let's plug this into the budget constraint:

$$2\,(3) + x_2 = 12$$

$$6 + x_2 = 12$$

$$x_2 = 6$$

$$(3, 6)$$

$u\,(3, 6) = 18$

2 An Aside on Equal Slope

2.1 Tradeoff Interpretation

MRS is the amount of x_2 you would be willing to give up to get a unit of x_1

Price ratio is the amount of x_2 you have to give up to get a unit of x_1.

If these aren't the same either I am willing to give up more x_2 to than I have to to get x_1, or I am willing to give up more x_1 than I have to get x_2. Either way, the bundle couldn't be optimal.

2.2 Bang-for-your-buck Interpretation

$$MRS = -\frac{mu_1}{mu_2}$$

$$-\frac{mu_1}{mu_2} = -\frac{p_1}{p_2}$$

$$\frac{mu_1}{mu_2} = \frac{p_1}{p_2}$$

$$\frac{mu_1}{p_1} = \frac{mu_2}{p_2}$$

$\frac{mu_1}{p_1}$ is how much extra utility you get by spending a dollar on good 1

$\frac{mu_2}{p_2}$ is how much extra utility you get by spending a dollar on good 2

Suppose those weren't the same then there is a way to redistribute your money and make yourself better of.