# 1 Preferences

## 1.1 MRS From Utility

$mu_1$ (marginal utility of good 1) measures how much utility increases when you increase $x_1$ by a little bit.

$mu_2$ (marginal utility of good 2) measures how much utility increases when you increase $x_2$ by a little bit.

$$mu_1 = 2mu_2$$

$$\frac{mu_1}{mu_2} = 2$$

$$MRS = -\frac{mu_1}{mu_2}$$

$mu_1$ how much does $u$ go up when we increase $x_1$?

$$mu_1 = \frac{\partial \left(u\left(x_1, x_2\right)\right)}{\partial x_1}$$

$$mu_2 = \frac{\partial \left(u\left(x_1, x_2\right)\right)}{\partial x_2}$$

## 1.2 Cobb Douglass

Preferences where you get tired of both goods.

If you have too much $x_2$ you would be willing to give up a lot of it to get some $x_1$.

If you have too much $x_1$ you would be willing to give up a lot of it to get some $x_2$.

$$u\left(x_1, x_2\right) = x_1^{\alpha} x_2^{\beta}$$

$$u\left(x_1, x_2\right) = x_1 x_2$$

$$MRS = -\frac{\frac{\partial(x_1 x_2)}{\partial x_1}}{\frac{\partial(x_1 x_2)}{\partial x_2}}$$

$$MRS = -\frac{\frac{\partial(x_1 x_2)}{\partial x_1}}{\frac{\partial(x_1 x_2)}{\partial x_2}} = -\frac{x_2}{x_1}$$

$u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$

$$MRS = -\frac{\frac{\partial\left(x_1^{\alpha} x_2^{\beta}\right)}{\partial x_1}}{\frac{\partial\left(x_1^{\alpha} x_2^{\beta}\right)}{\partial x_2}} = -\frac{\alpha}{\beta}\left(\frac{x_2}{x_1}\right)$$

$\alpha = 2, \beta = 1$ $u(x_1, x_2) = x_1^2 x_2^1$

$$MRS(x_1, x_2) = -\frac{2}{1}\frac{x_2}{x_1}$$

What is the slope of the indifference curve at the points $(10, 10)$?

$$MRS(10, 10) = -2$$

## 1.3 Quasi-Linear

A situation where you only get tired of one of the two goods. Ice cream and money.

Always have this form:

$u(x_1, x_2) = f(x_1) + x_2$

For examples:

$u(x_{1,} x_2) = ln(x_1) + x_2$

$mu_1 = \frac{\partial(\ln(x_1)+x_2)}{\partial x_1} = \frac{1}{x_1}$ (get tired of it)

$mu_2 = \frac{\partial(\ln(x_1)+x_2)}{\partial x_2} = 1$ (don't get tired of it)

$$MRS = -\frac{\frac{1}{x_1}}{1} = -\frac{1}{x_1}$$

Give us indifference curves that are parralell along vertical lines.

Willingness to trade-off between the goods only depends on the good that you get tired of $(x_1)$.

$u(x_1, x_2) = \sqrt{x_1} + x_2$

$mu_1 = \frac{\partial(\sqrt{x_1}+x_2)}{\partial x_1} = \frac{1}{2\sqrt{x_1}}$

$mu_2 = \frac{\partial(\sqrt{x_1}+x_2)}{\partial x_1} = 1$

$$MRS = -\frac{\frac{1}{2\sqrt{x_1}}}{1} = -\frac{1}{2\sqrt{x_1}}$$

# 2 Well Behaved Preferences

**Rational** preferences are **complete, transitive**

## 2.1 Monotonicity

*Monotonicity:* **Both things are good.**

$(x_1, x_2), (y_1, y_2)$

If you have at least as much of both goods, you like the result at least as much:

If $x_1 \geq y_1$ and $x_2 \geq y_2$ then:

$$(x_1, x_2) \succsim (y_1, y_2)$$

If $x_1 > y_1$ and $x_2 > y_2$ then:

$$(x_1, x_2) \succ (y_1, y_2)$$

For example between $(2, 1)$ and $(1, 0)$ both are strictly larger, and so:

$$(2, 1) \succ (1, 0)$$

For example between $(2, 1)$ and $(1, 1)$ both are strictly larger, and so:

All monotonicity requires here is that:

$$(2, 1) \succsim (1, 1)$$

This is allowed:

$$(2, 1) \sim (1, 1)$$

This is true, for instance, with perfect complements.

This is ruled out:

$$(1,1) \succ (2,2), (1,1) \sim (2,2)$$

Example, for these bundles $(2,1), (1,1)$ what is required:

$$(2,1) \succsim (1,1)$$

Example, for these bundles $(2,3), (1,1)$ what is required:

$$(2,3) \succ (1,1)$$

Is this allowed?

$$(1,1) \sim (1,2)$$

Yes.

Is this allowed?

$$(2,0) \succ (3,0)$$

No.

Is this allowed?

$$(3,3) \sim (2,2)$$

No.

Monotonicity does not require you to be strictly better off if you only get strictly more of one thing. It only requires you to be strictly better off if you get strictly more of **everything**.

Perfect complements are monotonic.

### 2.1.1 Monotonicity of Utility Function

We say a utility function is monotonic when:

For a pair of bundles $(x_1, x_2), (y_1, y_2)$

If $x_1 \geq y_1$ and $x_2 \geq y_2$ then:

$$u(x_1, x_2) \geq u(y_1, y_2)$$

If $x_1 > y_1$ and $x_2 > y_2$ then:

$$u(x_1, x_2) > u(y_1, y_2)$$

## 2.2 Examples

If we increase only one number and the utility function does not decrease, the first condition is met.

If we increase both numbers and the utility function strictly increases, the second condition is met.

$u(x_1, x_2) = x_1 + x_2$ is monotonic.

$u(x_1, x_2) = x_1 x_2$ is monotonic.

$u(x_1, x_2) = min\{x_1, x_2\}$ is monotonic.

$u(x_1, x_2) = ln(x_1) + x_2$ is monotonic.

$u(x_1, x_2) = x_1 - x_2$ is not monotonic.

$$u(2,1) = 1, u(2,2) = 0$$

$$(2,1) \succ (2,2)$$

This violates that monotonicity requires:

$$(2,2) \succsim (2,1)$$

### 2.2.1 Convexity

*Mixtures* are better.

$(2,0), (0,2)$

Mixtures:

Half of bowl one and mix it with half of bowl two we get a bowl with one scoop of each. $(1,1)$

$t = \frac{3}{4}$ of bowl one and $1 - t = \frac{1}{4}$ of bowl two: $(1.5, 0.5)$

***Convex combinations*** of the bundles.

A convex combination of $(x_1, x_2)$ and $(y_1, y_2)$ is a value $t \in [0,1]$.

If $t$ is the proportion of bundle $(x_1, x_2)$ then $(1-t)$ is the proportion of bundle $(y_1, y_2)$.

This results in the convex combination:

$$(tx_1 + (1-t) y_1, tx_2 + (1-t) y_2)$$

For example. $x = (2,0)$ and $y = (0,2)$

Then for $t = \frac{1}{2}$:

$$\left(\frac{1}{2}2 + \left(1 - \frac{1}{2}\right)0, \frac{1}{2}0 + \left(1 - \frac{1}{2}\right)2\right) = (1, 1)$$

$t = \frac{3}{5}$:

$$\left(\frac{3}{5}2 + \left(1 - \frac{3}{5}\right)0, \frac{3}{5}0 + \left(1 - \frac{3}{5}\right)2\right) = \left(\frac{6}{5}, \frac{4}{5}\right)$$

### 2.3 Definition of Convexity:

For two bundles $(x_1, x_2)$ and $(y_1, y_2)$ such that

$$(x_1, x_2) \sim (y_1, y_2)$$

Then for any $t \in [0, 1]$

$$(tx_1 + (1-t)\,y_1, tx_2 + (1-t)\,y_2) \succsim (x_1, x_2)$$

$$(tx_1 + (1-t)\,y_1, tx_2 + (1-t)\,y_2) \succsim (y_1, y_2)$$