1 Preferences

1.1 MRS From Utility

How to find the slope of an indifference cruve at a point using the utility function.

Marginal Utility. *How much does utility change when we change the amount of one of the goods by a little bit.*

Margainal utility of x_1 is mu_1

$$mu_1 = \frac{\partial\left(u\left(x_1, x_2\right)\right)}{\partial x_1}$$

Margainal utility of x_2 is mu_2

$$mu_2 = \frac{\partial\left(u\left(x_1, x_2\right)\right)}{\partial x_2}$$

MRS (the slope of the indifference curve at a point) measures how much good 2 I am willing to give up to get one more unit of good 1.

Suppose: $mu_1 = 10, mu_2 = 5.$

$$MRS = -2$$

$mu_1 = 2, mu_2 = 1$

$$MRS = -2$$

The slope of the indifference curve can be measured by the ratio of marginal utilitites.

$$MRS = -\frac{mu_1}{mu_2}$$

1.2 Examples

$u\left(x_1, x_2\right) = 3x_1 + 2x_2$

$$MRS = -\frac{mu_1}{mu_2} = -\frac{\frac{\partial(3x_1+2x_2)}{\partial x_1}}{\frac{\partial(3x_1+2x_2)}{\partial x_2}}$$

$$= -\frac{\frac{\partial(3x_1+2x_2)}{\partial x_1}}{\frac{\partial(3x_1+2x_2)}{\partial x_2}} = -\frac{3}{2}$$

1.2.1 Cobb Douglass

Cobb Douglass Preferences can be represented by utility functions that look like this:

$$u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$$

One example:

$$u(x_1, x_2) = x_1 x_2$$

$$MRS = -\frac{\frac{\partial(x_1 x_2)}{\partial x_1}}{\frac{\partial(x_1 x_2)}{\partial x_2}} = -\frac{x_2}{x_1}$$

More generally:

$$MRS = -\frac{\frac{\partial\left(x_1^{\alpha} x_2^{\beta}\right)}{\partial x_1}}{\frac{\partial\left(x_1^{\alpha} x_2^{\beta}\right)}{\partial x_2}}$$

$$\frac{\partial\left(x_1^{\alpha} x_2^{\beta}\right)}{\partial x_1} = \alpha x_1^{\alpha-1} x_2^{\beta}$$

$$\frac{\partial\left(x_1^{\alpha} x_2^{\beta}\right)}{\partial x_2} = \beta x_1^{\alpha} x_2^{\beta-1}$$

$$MRS = -\frac{\alpha x_1^{\alpha-1} x_2^{\beta}}{\beta x_1^{\alpha} x_2^{\beta-1}}$$

$x^2 x^5 = x^{2+5}$

$$MRS = -\frac{\alpha x_1^{\alpha} x_1^{-1} x_2^{\beta}}{\beta x_1^{\alpha} x_2^{\beta} x_2^{-1}} = -\frac{\alpha x_1^{-1}}{\beta x_2^{-1}} = -\frac{\alpha x_2}{\beta x_1}$$

$$MRS = -\frac{\alpha x_2}{\beta x_1}$$

1.2.2 Quasi-Linear

$u(x_1, x_2) = f(x_1) + x_2$

$$u(x_1, x_2) = ln(x_1) + x_2$$

$$mu_1 = \frac{\partial(ln(x_1) + x_2)}{\partial x_1} = \frac{1}{x_1}$$

$$mu_2 = 1$$

Ice cream and money.

$$MRS = -\frac{\frac{1}{x_1}}{1} = -\frac{1}{x_1}$$

$u(x_1, x_2) = \sqrt{x_1} + x_2$

$$MRS = -\frac{\frac{\partial(\sqrt{x_1}+x_2)}{\partial x_1}}{\frac{\partial(\sqrt{x_1}+x_2)}{\partial x_2}} = -\frac{1}{2\sqrt{x_1}}$$

2 Well Behaved Preferences

Rationality only requires **completeness** and **transitivity.**

2.1 Monotonicity

The assumption that you like both things.

Preferences are monotonic if the following two conditions are met:

(x_1, x_2), (y_1, y_2)

1. if $x_1 \geq y_1$ and $x_2 \geq y_2$ then:

$$(x_1, x_2) \succsim (y_1, y_2)$$

2. if $x_1 > y_1$ and $x_2 > y_2$ then:

$$(x_1, x_2) \succ (y_1, y_2)$$

What has to be true about these pairs?

$$(2,2) \succ (1,1)$$

Indifference is ruled out here.

$$(2,1) \succsim (1,1)$$

Does rule out indifference. $(2,1) \sim (1,1)$. It does rule $(1,1) \succ (2,1)$.

It puts no restriction on $(1,2)\,,(2,1)$

2.1.1 Monotonicity of Utility Function

A utility function is monotonic when:

1. if $x_1 \geq y_1$ and $x_2 \geq y_2$ then:

$$u\left(x_1, x_2\right) \geq u\left(y_1, y_2\right)$$

2. if $x_1 > y_1$ and $x_2 > y_2$ then:

$$u\left(x_1, x_2\right) > u\left(y_1, y_2\right)$$

Steps to check if a utility function is monotonic:

1. If you increase just one good, the utility doesn't decrease.

2. If you increase both goods, the utility increases.

2.2 Examples

u$(x_1, x_2) = x_1 + x_2$ is monotonic.

$u\left(x_1, x_2\right) = x_1 x_2$ is monotonic.

$u\left(x_1, x_2\right) = ln\left(x_1\right) + x_2$ is monotonic.

$u\left(x_1, x_2\right) = min\left\{x_1, x_2\right\}$ is monotonic.

$u\left(x_1, x_2\right) = x_1 - x_2$

$$(2,2) \succ (2,3)$$

$$(2,2) \succsim (2,3)\,,(2,3) \not\succsim (2,2)$$

2.2.1 Convexity

Mixtures are better than extremes.

$$(2, 0) \sim (0, 2)$$

$$(1, 1), (1.5, 0.5), (0.5, 1.5)$$

Convex Combinations of the bundles.

$$(1, 1) \succsim (2, 0)$$

$$(1, 1) \succsim (0, 2)$$