1 Some Review

1.1 Example of question on relation properties.

Has the same eye color as.

Is this a reflexive relation?

Yes.

Is complete?

No.

Is it transitive?

Yes.

Assymetric?

No. If it holds in one direction, it holds in borth directions.

Symetric

Yes. If it holds in one direction, it holds in borth directions.

1.2 Example of question on relation properties.

For the set $X = \{a, b, c\}$ consider the following preference relation:

$$a \succsim a, b \succsim b, c \succsim c, a \succsim b, b \succsim c, c \succsim a$$

1. Is this reflexive?

Yes.

2. Is this complete?

Yes.

3. Provide an example of why this relation is not transitive.

Since $a \succsim b, b \succsim c$ but $a \succsim c$ does not appear, this is **not transitive**.

1.3 Other Relation Stuff

For the following complete and transitive preference relation:

$$a \succsim a, b \succsim b, c \succsim c, a \succsim b, b \succsim a, a \succsim c, b \succsim c$$

What is the $\succ$.

$$a \succ c, b \succ c$$

What is the $\sim$.

$$a \sim a, b \sim b, c \sim c, a \sim b$$

Write the preference relation in chain notation.

$$a \sim b \succ c$$

$$b \sim a \succ c$$

$$a \succsim a, b \succsim b, c \succsim c, a \succsim b, b \succsim a, a \succsim c, b \succsim c$$

What is best from the set $\{a, b, c\}$?

1.4 Using an Utility Function to Find Indifference Curves

$$u\left(x_1, x_2\right) = 2x_1 + x_2$$

Plot a few indifference curves. Here, we can use the MRS:

$$MRS = -\frac{2}{1} = -2$$

All of the indifference curves have slope -2. They are all stright lines.

1.4.1 Another Plotting Question

Plot the set of bundles indifferent to $(5, 5)$.

This is the set of bundles same utility as $(5, 5)$.

$$u\left(5, 5\right) = 2 * 5 + 5 = 15$$

All of the bundles that give utility 15 are given by:

$$2x_1 + x_2 = 15$$

$$x_2 = 15 - 2x_1$$

1.5 Plotting Perfect Compelments Indiffernce Curves

Perfect Complements:

$$min\left\{\frac{1}{2}x_1, x_2\right\}$$

Line of no waste. (This line connects the kinks of each indiffence curve):

$$x_2 = \frac{1}{2}x_1$$

Let's find some bundles indifferent to $(2, 1)$:

$u(2,1) = min\left\{\frac{1}{2}2, 1\right\} = 1$

$u(2,1) = 1$

$u(3,1) = 1$

$u(10,1) = 1$

$u(2,2) = 1$

Thus, the indifference cuve is L-shaped with a kink at $(2, 1)$.

2 Representing Preferences

A utility function represents preferences if any time $u(x) \geq u(y)$ then $x \succsim y$ and vise versa.

$$a \succsim a, b \succsim b, c \succsim c, a \succsim b, b \succsim a, a \succsim c, b \succsim c$$

In chain notation:

$$a \sim b \succ c$$

One utility representation:

$$u(a) = 1, u(b) = 1, u(c) = 0$$

Another:

$$u(a) = 2, u(b) = 2, u(c) = 0$$

2.1 Preferences From Utility

Given a utility function:

$$u(a) = 5, u(b) = 1, u(c) = 3$$

Write the preference relation in chain notation:

$$a \succ c \succ b$$

In terms of the weak preference relation, this is the same as:

$$a \succsim a, b \succsim b, c \succsim c, a \succsim c, a \succsim b, c \succsim b$$

3 Finding MRS

3.1 Perfect Substitutes

$$u(x_1, x_2) = 3x_1 + 2x_2$$

$$MRS = -\frac{3}{2}$$

3.2 Cobb Douglass

$$u(x_1, x_2) = x_1^2 x_2^1$$

$$MU_1 = \frac{\partial \left(x_1^2 x_2\right)}{\partial x_1} = 2x_1 x_2$$

$$MU_2 = \frac{\partial \left(x_1^2 x_2\right)}{\partial x_2} = x_1^2$$

$$MRS = -\frac{2x_1 x_2}{x_1^2} = -2\frac{x_2}{x_1}$$

3.3 Quasi-Linear

$$u(x_1, x_2) = ln(x_1) + x_2$$

$$Mu_1 = \frac{1}{x_1}$$

$$MU_2 = 1$$

$$MRS = -\frac{\frac{1}{x_1}}{1} = -\frac{1}{x_1}$$

4 Monotonicity

More is better than less.

Formally:

If $x_1 \geq y_1$ and $x_2 \geq y_2$ then $(x_1, x_2) \succsim (y_1, y_2)$

If $x_1 > y_1$ and $x_2 > y_2$ then $(x_1, x_2) \succ (y_1, y_2)$

Find a pair of bundles that shows this utility function is not monotonic:

$$u(x_1, x_2) = x_1 - 5x_2$$

$$(1, 1) \succ (1, 2)$$

However since $(1, 2)$ has at least as much of everything, we need $(1, 2) \succsim (1, 1)$.

5 Convexity

$(2, 0)$ $(0, 2)$ mixing these together with $t = \frac{1}{2}$ weight on $(2, 0)$ gives the convex combination $(1, 1)$.

Convexity: for any two bundles that are indifferent, a mixture is at least as good as both. In this case, if $(2, 0) \sim (0, 2)$ then $(1, 1) \succsim (2, 0)$ and $(1, 1) \succsim (0, 2)$

6 Three Possibilites for Finding Demand

1. **If the utility is smooth (cobb douglass/quasi-linear)**

Tangency: $mrs = -\frac{p_1}{p_2}$

Budget: $p_1x_1 + p_2x_2 = m$

2. **Perfect Substitutes.**

One of the endpoints of the budget will be optimal.

3. **Perfect Complements:** $min\left\{ax_1, bx_2\right\}$

No-Waste: $ax_1 = bx_2$

Budget: $p_1x_1 + p_2x_2 = m$