1 Market Demand and Elasticity

1.1 Market Demand

For someone's who's utility function $u(x_1, x_2) = x_1 x_2$. (Cobb Douglass)

Invidiual demands. $x_1 = \frac{\frac{1}{2}m}{p_1}$

Market is made up of many individuals.

To write the individual demands we need to indicate which good it is (number) and for which person (letter):

$x_{1,a}$ demand for person a of good 1.

$x_{2,a}$ demand for person a of good 2.

$x_{1,b}$ demand for person b of good 1.

X_1 Market demand for good 1.

X_2 Market demand for good 2.

To get market demand, we sum individual demands. If there are 3 consumers $\{a, b, c\}$.

$$X_1 = x_{1,a} + x_{1,b} + x_{1,c}$$

More generally, if the set of people is $P = \{a, b, c, d, ...\}$

$$X_1 = \sum_{i \in P} x_{1,i}$$

1.2 Example: Cobb Douglass

Let m_i is the income of person i.

If we have three consumers with each with utility function $U(x_1, x_2) = x_1 x_2$ and incomes $m_a = 10, m_b = 20, m_c = 30$, demands are:

$$x_{1,a} = \frac{\frac{1}{2}m_a}{p_1}, x_{1,b} = \frac{\frac{1}{2}m_b}{p_1}, x_{1,c} = \frac{\frac{1}{2}m_c}{p_1}$$

The market demand for 1 is:

$$X_1(p_1, p_2, m_a, m_b, m_c) = \frac{\frac{1}{2}m_a}{p_1} + \frac{\frac{1}{2}m_b}{p_1} + \frac{\frac{1}{2}m_c}{p_1}$$

$$= \frac{1}{2}\frac{1}{p_1}(m_a + m_b + m_c)$$

1.3 Example with Quasi-Linear Utility

Three consumers with utility: $U\left(x_1, x_2\right) = \ln\left(x_1\right) + x_2$

Incomes are $m_a = 10, m_b = 20, m_c = 30$

What is each consumer's marshallian demand?

$$U\left(x_1, x_2\right) = \ln\left(x_1\right) + x_2$$

Tangency condition:

$$MRS = -\frac{p_1}{p_2}$$

$$MU_1 = \frac{\partial\left(\ln\left(x_1\right) + x_2\right)}{\partial x_1} = \frac{1}{x_1}$$

$$MU_2 = \frac{\partial\left(\ln\left(x_1\right) + x_2\right)}{\partial x_2} = 1$$

$$MRS = -\frac{\frac{1}{x_1}}{1} = -\frac{1}{x_1}$$

Tangency condition:

$$-\frac{1}{x_1} = -\frac{p_1}{p_2}$$

$$x_1 = \frac{p_2}{p_1}$$

Budget constaint:

$$p_1 x_1 + p_2 x_2 = m$$

$$p_1\left(\frac{p_2}{p_1}\right) + p_2 x_2 = m$$

$$p_2 + p_2 x_2 = m$$

$$p_2 (1 + x_2) = m$$

$$x_2 = \frac{m}{p_2} - 1$$

Market demand for x_1

$x_{1,a} = \frac{p_2}{p_1}, x_{1,b} = \frac{p_2}{p_1}, x_{1,c} = \frac{p_2}{p_1}$

$$X_1 = \frac{p_2}{p_1} + \frac{p_2}{p_1} + \frac{p_2}{p_1} = 3\frac{p_2}{p_1}$$

Market demand for good 2:

$$X_2 = \frac{m_a}{p_2} - 1 + \frac{m_b}{p_2} - 1 + \frac{m_c}{p_2} - 1$$

$$X_2 = \frac{30}{p_2} - 1 + \frac{20}{p_2} - 1 + \frac{10}{p_2} - 1$$

$$X_2 = \frac{60}{p_2} - 3$$

2 Elasticity

The point elasiticity is to measure relationships between variables in purely percentage terms. This allows to compare behavior of demand between markets.

If the price goes up by 1% and demand decreases by 1% in two different markets, they have the same kind of behavior even if they have wildly different magnitudes for prices or quantities.

2.1 Price Elasticity- Definition

Regular change in demand when price changes:

$$\frac{\Delta x}{\Delta p}$$

Demand goes down by 10 when price went up by 2 dollars. Then it goes down by 5 units per dollar:

$$-\frac{10}{2} = -5$$

The **percent change in demand** is $\frac{\Delta x}{x}$. Suppose demand is 100 and goes up by 10. That's a 10% increase.

$$\frac{\Delta x}{x} = \frac{10}{100} = .1 = 10\%$$

The **percent change in price** is $\frac{\Delta p}{p}$. Price changes from 1.00 to 1.10, that's a 10% increase.

$$\frac{\Delta p}{p} = \frac{0.10}{1} = 0.1 = 10\%$$

The relationship between demand and price in purely percentage terms is the ratio of these:

$$\frac{\%\Delta x}{\%\Delta p} = \frac{\frac{\Delta x}{x}}{\frac{\Delta p}{p}} = \frac{\Delta x}{\Delta p}\frac{p}{x}$$

To turn this into an elasticity, turn the finite changes Δ into very small changes ∂. We get this equation for the price elasticity.

$$\epsilon_{1,1} = \frac{\partial x}{\partial p}\frac{p}{x}$$

2.2 Interpreting Price Elasticity

Suppose $\epsilon_{1,1} = -1$

When the price of the good goes up by 1%, demand will decrease by 1%.

2.2.1 Classifications

For all ordinary elasticity will be negative.

$|\epsilon_{1,1}| > 1$ ($\epsilon_{1,1} = -50$) price goes up by 1%, demand goes down by 50%. Very sensitive demand. **Elastic**.

$|\epsilon_{1,1}| < 1$ $\left(\epsilon_{1,1} = -\frac{1}{10}\right)$ price goes up by 1% demand goes down by 0.1%. Not sensitive to changes in price. **Inelastic.**

$|\epsilon_{1,1}| = 1$

2.3 Income Elasticity

η measures how demand for a good changes in percentage terms when income increases by 1% .

$$\eta = \frac{\partial x}{\partial m}\frac{m}{x}$$

$$\eta = 1$$

Income goes up by 1% their demand goes up by 1%.

2.4 Interpreting Elasticities Examples

Suppose $\epsilon_{1,1} = -2$. The price of a good increases by 1%, what happens to their demand for x_1?

Suppose $\eta = 0.5$. Income increases by 1%, what happens to their demand for x_1?

$\epsilon_{1,2}$ is the **cross-price-elasticity** for good 1 when p_2 changes.

Suppose $\epsilon_{1,2} = 2$. The price of a good 2 increases by 1%, what happens to their demand for x_1?

Goes up by 2%. These goods are substitutes!

2.4.1 Example 1

A consumer has $u(x_1, x_2) = x_1 x_2$. Their demand is for x_1 is:

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

$$\epsilon_{1,1} = \frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1}$$

$$= \frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial p_1}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}}$$

$$\frac{\partial\left(\frac{1}{2}mp_1^{-1}\right)}{\partial p_1}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}}$$

$$= -\frac{1}{2}mp_1^{-2}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}}$$

$$= -\frac{1}{2}mp_1^{-2}\frac{p_1}{1}\frac{p_1}{\frac{1}{2}m}$$

$$= -\frac{1}{2}m\frac{1}{p_1^2}\frac{p_1^2}{\frac{1}{2}m}$$

$$= -\frac{1}{2}\frac{1}{\frac{1}{2}} = -1$$

For a 1% increase in price, demand goes down by 1%.

$$\frac{\partial (x_1)}{\partial m}\frac{m}{x_1} = \frac{\partial \left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial m}\frac{m}{\frac{\frac{1}{2}m}{p_1}}$$

$$= \frac{\partial \left(\frac{\frac{1}{2}}{p_1}m\right)}{\partial m}\frac{m}{\frac{\frac{1}{2}m}{p_1}}$$

$$= \frac{\frac{\frac{1}{2}m}{p_1}}{\frac{\frac{1}{2}m}{p_1}} = 1$$

2.4.2 Example 2

A consumer has $u(x_1, x_2) = min\{x_1, x_2\}$

$$x_1 = \frac{m}{p_1 + p_2}$$

$\eta = \frac{\partial (x_1)}{\partial m}\frac{m}{x_1}$
$\eta = 1$

$$= \frac{m}{p_1 + p_2} \frac{1}{\frac{m}{p_1+p_2}} = 1$$

$$\epsilon_{1,1} = \frac{\partial \left(\frac{m}{p_1+p_2} \right)}{\partial p_1} \frac{p_1}{\frac{m}{p_1+p_2}}$$