1 Elasticity Problems

1.1 Elasticity Problem 1

$U(x_1, x_2) = min\{x_1, x_2\}$

$x_1 = \frac{m}{p_1+p_2}$

1.1.1 Income Elasticity of Demand

Measures the percent relationship between demand and income.

$\eta = \frac{\partial x_1}{\partial m}\frac{m}{x_1}$

We want all of our elasticities to be in terms of p_1, p_2, m.

$$\frac{\partial\left(\frac{m}{p_1+p_2}\right)}{\partial m}\frac{m}{\frac{m}{p_1+p_2}}$$

Another way to write demand:

$$x_1 = \frac{1}{p_1+p_2}m$$

$$\frac{\partial(x_1)}{\partial m} = \frac{1}{p_1+p_2}$$

$$\eta = \frac{1}{p_1+p_2}\frac{m}{\frac{m}{p_1+p_2}} = \frac{\frac{m}{p_1+p_2}}{\frac{m}{p_1+p_2}} = 1$$

A one percent increase in income will lead to a one percent increase in demand.

Calculate the price elasticity of demand.

$$\epsilon_{1,1} = \frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1}$$

$$\epsilon_{1,1} = \frac{\partial\left(\frac{m}{p_1+p_2}\right)}{\partial p_1}\frac{p_1}{\frac{m}{p_1+p_2}}$$

$$\frac{\partial\left(\frac{m}{p_1+p_2}\right)}{\partial p_1} = \frac{\partial\left(m(p_1+p_2)^{-1}\right)}{\partial p_1}$$

$$-1m\left(p_1+p_2\right)^{-2}(1)=-m\left(p_1+p_2\right)^{-2}=-\frac{m}{\left(p_1+p_2\right)^2}$$

$$\frac{\partial\left(x_1\right)}{\partial p_1}\frac{p_1}{\frac{m}{p_1+p_2}}=-\frac{m}{\left(p_1+p_2\right)^2}\frac{p_1}{\frac{m}{p_1+p_2}}$$

$$-\frac{m}{\left(p_1+p_2\right)\left(p_1+p_2\right)}\frac{p_1}{\frac{m}{p_1+p_2}}$$

$$-\frac{1}{\left(p_1+p_2\right)}\frac{m}{\left(p_1+p_2\right)}\frac{p_1}{\frac{m}{p_1+p_2}}$$

$$-\frac{1}{\left(p_1+p_2\right)}\frac{p_1}{}=-\frac{p_1}{p_1+p_2}$$

Suppose $p_1=p_2=1$

$$\epsilon_{1,1}=-\frac{1}{1+1}=-\frac{1}{2}$$

When the price of x_1 goes up by 1% demand goes down by $\frac{1}{2}$%.

Since $\frac{p_1}{p_1+p_2}<1$ demand will always be inelastic for perfect this demand function.

1.1.2 Elasticticy Problem 2

One hundred consumers have individual demand for some good x of $x=10-p$.

a. What is the individual price elasticity of demand for this good? (This will be a function of p).

$$\frac{\partial x}{\partial p}\frac{p}{x}=\frac{\partial x}{\partial p}\frac{p}{10-p}=\frac{\partial\left(10-p\right)}{\partial p}\frac{p}{10-p}=-1\frac{p}{10-p}$$

$$=-\frac{p}{10-p}$$

b. When $p=9$ what is individual price elasticity of demand?

$$=-\frac{9}{10-9}=-\frac{9}{1}=-9$$

c. When $p = 9$ what happens to individual demand when price increases by 1%?

Demand decreases by 9%. This is an elastic good.

(An aside, if $\epsilon_{1,1} = -1$, we say the good is "unit elastic").

d. What is the **market demand** for this good?

100 consumers each have $10 - p$.

$$X = 100\left(10 - p\right) = 1000 - 100p$$

e. What is the market price elasticity of demand for this good? (This will be a function of p).

$$= \frac{\partial\left(1000 - 100p\right)}{\partial p}\frac{p}{X}$$

$$= -100\frac{p}{1000 - 100p} = -\frac{100p}{100\left(10 - p\right)}$$

$$= -\frac{p}{10 - p}$$

2 Exam Review

$$MRS = -\frac{p_1}{p_2}$$

$$-\frac{3}{2} = -1$$

$$U\left(20, 0\right) = 3 * 20 + 2 * 0 = 60$$

Part *d*. $(10, 0)\,(0, 20)$

$$U\left(10, 0\right) = 3 * 10 + 2 * 0 = 30$$

$$U(0,20) = 3*0 + 2*20 = 40$$

$$(0,20)$$

$$U(0,20) = 3*0 + 2*20 = 40$$

$(20,0)$

How much to buy the old bundle at the new prices?

$$(20,0)$$

$2*20 + 1*0 = 40$

$\tilde{m} = 40$

What do they buy at this income but with the new prices?

$p_1 = 2, p_2 = 1, \tilde{m} = 40$

$$(20,0),(0,40)$$

$U(20,0) = 3*20 + 2*0 = 60, U(0,40) = 3*0 + 2*40 = 80$

$$(0,40)$$

$0 - 20 = -20$

Because the substitution effect is the whole effect (20) ,the income effect is 0.

2.1 Problem 4

$r = 0.5, m_1 = 1000, m_2 = 2000$

$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$

$$c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}$$

$1.5c_1 + c_2 = 1.5(1000) + 2000$

$1.5c_1 + c_2 = 3500$

If you only consumer in period 2, you can have 3500.

$u = min\{c_1, c_2\}$

$c_1 = c_2$

$1.5c_1 + c_2 = 3500$

$1.5c_1 + c_1 = 3500$

$2.5c_1 = 3500$

$c_1 = 1400, c_2 = 1400$

$m_1 = 1000, m_2 = 2000$

Borrower since $c_1 > m_1$

$U(2, 4) = 2 * 4 = 8$

What is another bundle with utility 8?

$(1, 8), (8, 1)$ $U(8, 1) = 8 * 1 = 8$

$(3, 3)$

$(3, 3) \succsim (2, 4)$

$U(3, 3) = 9, U(2, 4) = 8$