1 Elasticity Continued

Price elasticity. How much does demand change when the price of that good changes.

$\epsilon_{1,1} = -1$ when p_1 goes up by 1%, demand *decreases* by 1%.

$\epsilon_{1,2} = 1$ when p_2 goes up by 1%, demand *increases* by 1%.

$\eta = 1$ (income elasticity) when (income) m increaes by 1%, demand increases by 1%.

1.1 Elasticity Formula

How does demand change when price changes.

Price went from 10 to 11. Demand went down from 100 to 50.

$$\frac{-50}{1} = \frac{\Delta x}{\Delta p}$$

What is the percent change in price?

$$\% \Delta p? = 10\%$$

$$\frac{\Delta p}{p} = \frac{11 - 10}{10} = \frac{1}{10} = 0.1 = 10\%$$

$$\frac{\Delta x}{x} = \frac{-50}{100} = -\frac{1}{2} = -50\%$$

$$\frac{\frac{\Delta x}{x}}{\frac{\Delta p}{p}} = \frac{-50\%}{10\%} = -5\%$$

$\frac{\frac{\Delta x}{x}}{\frac{\Delta p}{p}}$ elasticity measure for "finite differences"

$\frac{\frac{\partial x}{x}}{\frac{\partial p}{p}} = \frac{\partial x}{\partial p} \frac{p}{x}$

Price elasticity of good 1 when we change p_1:

$$\epsilon_{1,1} = \frac{\partial x_1}{\partial p_1} \frac{p_1}{x_1}$$

Cross price elasticity between x_1 and p_2 :

$$\epsilon_{1,2} = \frac{\partial x_1}{\partial p_2}\frac{p_2}{x_1}$$

What is the formula for the income elasticity of good x_1:

$$\eta = \frac{\partial x_1}{\partial m}\frac{m}{x_1}$$

1.1.1 Elasticity Problem 1

$U(x_1, x_2) = x_1 x_2$

$$x_1 = \frac{\frac{1}{2}m}{p_1}$$

Income elasticity. Relationship between x_1 and m.

$$\eta = \frac{\partial x_1}{\partial m}\frac{m}{x_1}$$

We always want an elasticity to be in terms of variables p_1, p_2, m or just a number.

Step 1, plug in demand:

$$\eta = \frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial m}\frac{m}{\frac{\frac{1}{2}m}{p_1}}$$

Step 2, evaluate the derivative:

$$\eta = \frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial m}\frac{m}{\frac{\frac{1}{2}m}{p_1}}$$

$$\frac{\partial\left(\frac{1}{2}\frac{1}{p_1}m\right)}{\partial m} = \frac{1}{2}\frac{1}{p_1}$$

Step 3: Simplify if you can.

$$\eta = \frac{1}{2}\frac{1}{p_1}\frac{m}{\frac{\frac{1}{2}m}{p_1}} = \frac{1}{p_1}\frac{\frac{1}{2}m}{\frac{\frac{1}{2}m}{p_1}} = \frac{\frac{\frac{1}{2}m}{p_1}}{\frac{\frac{1}{2}m}{p_1}} = 1$$

Price elasticity for this demand:

$x_1 = \frac{\frac{1}{2}m}{p_1}$

$$\epsilon_{1,1} = \frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1}$$

Step 1, plug in the demand:

$$\epsilon_{1,1} = \frac{\partial\left(\frac{\frac{1}{2}m}{p_1}\right)}{\partial p_1}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}}$$

$$\frac{\partial\left(\frac{1}{2}mp_1^{-1}\right)}{\partial p_1} = -1\frac{1}{2}mp_1^{-2} = -\frac{1}{2}\frac{m}{p_1^2}$$

$$= -\frac{1}{2}\frac{m}{p_1^2}\frac{p_1}{\frac{\frac{1}{2}m}{p_1}} = -\frac{1}{2}\frac{1}{p_1^2}\frac{p_1}{\frac{\frac{1}{2}}{p_1}} = -\frac{1}{p_1^2}\frac{p_1^2}{} = -1$$

$$\epsilon_{1,1} = -1$$

Income Elasticity of Demand

Elasticticy Problem 2 individual demand for some good x_1 of $x_1 = 10 - p_1$.

$$\epsilon_{1,1} = \frac{\partial\left(10 - p_1\right)}{\partial p_1}\frac{p_1}{10 - p_1}$$

$$= (-1)\frac{p_1}{10 - p_1} = -\frac{p_1}{10 - p_1}$$

At the price $p_1 = 9$ what is the price elasticity of demand:

$$-\frac{9}{10-9} = -\frac{9}{1} = -9$$

This is an elastic good when $p_1 = 9$ and when the price of p_1 goes up by 1%, demand will go down by 9%.

Suppose $p_1 = 1$. How much does demand change in percent terms when the price goes up by 1%?

$$-\frac{p_1}{10-p_1} = -\frac{1}{9}$$

This an inelastic good when $p_1 = 1$ and when price goes up by 1%, demand goes down by $\frac{1}{9}$%.

2 Exam Review

Tangency

$$MRS = -\frac{p_1}{p_2}$$

$$-\frac{x_2}{x_1} = -\frac{p_1}{p_2}$$

$3x_1 + 2x_2$

$$-\frac{3}{2} = -\frac{1}{1}$$

For any perfect substitutes problem, one of the endpoints will be optimal.

When $p_1 = 2, p_2 = 1$

$$(10, 0), (0, 20)$$

$$u(10, 0) = 3 * 10 + 2 * 0 = 30$$

$$u(0, 20) = 3 * 0 + 2 * 20 = 40$$

2.1 Intertemporal Choice

c_1 will always be more expensive than c_2.

$$(1+r)\,c_1 + c_2 = (1+r)\,m_1 + m_2$$

$$-\,(1+r)$$

$$(1+0.5)\,c_1 + c_2 = 1.5\,(1000) + 2000$$

$$1.5c_1 + c_2 = 1500 + 2000$$

$$1.5c_1 + c_2 = 3500$$

$min\,\{c_1, c_2\}$.

At the optimum, **no waste condition.**

$$c_1 = c_2$$

$$1.5c_1 + c_2 = 3500$$

$$1.5c_2 + c_2 = 3500$$

$$2.5c_2 = 3500$$

$$c_2 = \frac{3500}{2.5} = 1400$$

$$c_1 = 1400$$

$$(1400, 1400)$$

Borrower.

If you borrower and the interest rate goes down, you remain a borrower.