1 Equilibrium with Taxes

$Q_d(p) = 200 - 40p, Q_s(p) = 10p$

p^* is the price where demand is equal to supply.

$$200 - 40p = 10p$$

$$200 = 50p$$

$$p^* = 4$$

$$Q^* = 40$$

The inverse demand:

$$Q = 200 - 40p$$

$$40p = 200 - Q$$

$$p = 5 - \frac{1}{40}Q$$

The inverse supply:

$$Q = 10p$$

$$p = \frac{1}{10}Q$$

Consumer Surplus:

$$CS = \frac{1}{2}(40)(1) = 20$$

$$PS = \frac{1}{2}(40)(4) = 80$$

$$TS = 20 + 80 = 100$$

1.1 Adding a Tax

We want to add a quantity tax to the market. t per unit purchased.

Consumers will pay $p + t$

Firm only receive p

Goverment receive t

Suppose $p = 2$ and $t = 1$. Consumer pay 3 per unit, firms receive 2 per unit and the government receive 1 per unit.

What is an equilibrium with a tax?

What do consumers demand?

$$Q_d(p + t)$$

What do firm produce?

$$Q_s(p)$$

Motivating a new equilibrium condition:

Suppse $Q_d(p + t) > Q_s(p)$, there is an over-demand for the good and price will tend to rise.

Suppse $Q_d(p + t) < Q_s(p)$, there is an over-supply for the good and price will tend to fall.

In equilibrium:

$$Q_d(p + t) = Q_s(p)$$

1.2 Back to Example

$Q_d(p) = 200 - 40p, Q_s(p) = 10p$

Impose a quantity tax of $t = \frac{5}{2}$

What happens if the price doesn't adjust (it stays at $p^* = 4$):

$$Q_d\left(4 + \frac{5}{2}\right) = 200 - 40\left(4 + \frac{5}{2}\right) = -60$$

The fact that demand is negative here shows us tht the demand is zero at this price. No consumer is willing to spend 6.5 on the good.

Clearly, firms will need to lower their price p in order to sell any units at all.

$$200 - 40\left(p + \frac{5}{2}\right) = 10p$$

$$200 - 40p - 40\frac{5}{2} = 10p$$

$$200 - 100 = 50p$$

$$100 = 50p$$

$$p^* = 2$$

The firm supply at $p = 2$ is $10 * 2 = 20$

The consumer demand at $p = 2$ with a tax of $t = \frac{5}{2}$ is:

$$200 - 40\left(2 + \frac{5}{2}\right) = 200 - 180 = 20$$

1.3 Graphically

Notice that the difference between what consumer pay $p + t$ and what firms receive p is exactly the tax t.

In our example:

$$CS = \frac{1}{2}\left(\frac{1}{2}\right)(20) = 5$$

$$PS = \frac{1}{2}(2)(20) = 20$$

$$G = \frac{5}{2} * 20 = 50$$

$$TS = 75$$

The deadweight loss is the difference between the old total surplus and the new total surplus:

$$DWL = 25$$

1.4 Example

$Q_d(p) = 300 - 2p$

$Q_s(p) = p$

a) What is the equilibrium price and quantity?

$$300 - 2p = p$$

$$300 = 3p$$

$$p = 100$$

$$Q_s(100) = 100, Q_d(100) = 300 - 2(100) = 100$$

$$Q^* = 100$$

Inverse demand and supply:

$$Q = 300 - 2p$$

$$2p = 300 - Q$$

$$p = 150 - \frac{1}{2}Q$$

Inverse supply:

$$Q = p$$

$$p = Q$$

b) What are the consumer and producer surplus in equibrlium?

$$CS = \frac{1}{2} * 50 * 100 = 2500$$

$$PS = \frac{1}{2} * 100 * 100 = 5000$$

$$TS = 7500$$

c) What is the equilium price and quanity with a tax $t = 75$

$$300 - 2(p + 75) = p$$

$$300 - 2p - 2 * 75 = p$$

$$300 - 150 = 3p$$

$$150 = 3p$$

$$50 = p^*$$

Firm's receive 50, consumers pay $50 + 75 = 125$.

$$Q_s(50) = 50$$

$$Q_d(50 + 75) = 300 - 2(50 + 75) = 300 - 250 = 50$$

d) What is the consumer, prducer surplus, the government revenue, and the deadweight loss under this tax.

$$CS = \frac{1}{2}(50)(25) = 625$$

$$PS = \frac{1}{2}(50)(50) = 1250$$

$$G = 50 * 75 = 3750$$

$$TS = 625 + 1250 + 3750 = 5625$$

Old total surplus was 7500

$$DWL = 7500 - 5625 \quad = \quad 1875$$

We can also calculate DWL directly:

$$\frac{1}{2} * 75 * 50 \quad = \quad 1875$$

1.4.1 Incidence of a Tax

The original price was 100.

Consumer pay 125 after the tax is imposed.

Consumer pay 25 more than they used to.

Firms receive 50 after the tax is imposed.

Firms receive 50 less than they used to.

$25 + 50 = 75$

50 of the tax falls on producers and 25 falls on consumers.

This is called calculating the *incidence* of a tax. **Tax burden**.

Whichever side of the market is reltively more inelastic will bear a majority of the tax burder.

2 Enrichment: Maximizing Government Revenue

(This is not testable content, but is a response to a question received in class.)

How would the goverment find the t that maxzimizes government revenue.

First, let's calculate the equilibrium price and quantity as a function of the tax.

$$300 - 2\,(p + t) = p$$

$$300 - 2p - 2t = p$$

$$300 - 2t = 3p$$

$$100 - \frac{2}{3}t = p^*$$

$$Q\left(100 - \frac{2}{3}t\right) = 100 - \frac{2}{3}t$$

$$Q^* = 100 - \frac{2}{3}t$$

Government Revenue:

$$tQ^* = t\left(100 - \frac{2}{3}t\right)$$

$$G(t) = t\left(100 - \frac{2}{3}t\right)$$

$$100t - \frac{2}{3}t^2$$

Maximized where:

$$\frac{\partial\left(100t - \frac{2}{3}t^2\right)}{\partial t} = 0$$

$$100 - \frac{4}{3}t = 0$$

$$100 = \frac{3}{4}t$$

$$t = 75$$