1 Cost Min

$$\pi\left(x_1, x_2\right) = pf\left(x_1, x_2\right) - w_1 x_1 - w_2 x_2$$

For example if production is cobb-dougalss, we might have:

$$\pi\left(x_1, x_2\right) = p x_1^{\frac{1}{4}} x_2^{\frac{1}{4}} - w_1 x_1 - w_2 x_2$$

This can be tedious to maximize.

Profit maximization implies cost minimiztion.

If a firm is not producing their output in the cheapest possible way, they can change the way the produce that output. Their revenue will remain the same, but their costs will go down, and this will increase profit.

1.1 Two-Step Process

1. (Today) Figure out how to produce **any** level of output y in the cheapest possible way. Find the cost associate with that. $c\left(y\right)$.

2. Set up a profit fuction that only depends on output y. $\pi\left(y\right) = p\left(y\right) y - c\left(y\right)$

1.2 The Goal

The cost of cost minimization is to figure out the optimal way of producing any level of output y.

Prices of intpus w_1, w_2

Production function $f\left(x_1, x_2\right)$

Firm Goal: find a bundle $\left(x_1, x_2\right)$ that minimizes cost $w_1 x_1 + w_2 x_2$ subject to the constraint that the bundle produces y units of output $f\left(x_1, x_2\right) = y$.

Consumer Goal: find a bundle $\left(x_1, x_2\right)$ that maximIes utility u(x_1, x_2) subject to the constraint that the bundle is afford able $p_1 x_1 + p_2 x_2 = m$.

Dual Consumer Problem: find a bundle $\left(x_1, x_2\right)$ tht minimizes cost $p_1 x_1 + p_2 x_2$ of achieving some utility $u\left(x_1, x_2\right) = u$.

1.3 Isocosts and Isoquants

Another way to think about the firm problem is that we are looking for the bundle on the **isoquant** $f\left(x_1, x_2\right) = y$ that is also on the lowerst **isocost** curve.

Isocost curves are lines that represent sets of bundles that cost the same.

$$w_1x_1 + w_2x_2 = c$$

Slope of these are $-\frac{w_1}{w_2}$.

1.4 Three Possibilities

1. Tangency $TRS = -\frac{w_1}{w_2}$

Cobb-Douglass, or any utility function you can derivatives of.

2. No Waste Condition

Perfect Complements

3. One of the two endpoints is optima (use all x_1 or all x_2)

Perfect Substitutes

1.4.1 Interpreting the Tangency Condition

The geometry of the tangency condition is that if two curves meet eachother and don't have the same slope, they form an X shape and cross through eachother.

The economics:

$$-\frac{MP_1}{MP_2} = -\frac{w_1}{w_2}$$

$$\frac{w_2}{MP_2} = \frac{w_1}{MP_1}$$

$\frac{MP_1}{w_1}$ extra output you get by spending 1 more dollar on x_1.

$\frac{w_1}{MP_1}$ cost of increasing output by 1 unit using x_1.

Suppose

$$\frac{w_2}{MP_2} > \frac{w_1}{MP_1}$$

In this case, I can decrease x_2 and increase x_1 in way tht produces the same out but, but it the net effect is a cost savings.

If the producivity per dollar of each input is not the same, then there is a way to reallocate inputs and save money while producing the same output.

1.4.2 An Example

$w_1 = 1, w_2 = 1$

$f(x_1, x_2) = 2x_1 + x_2$

Use all x_1. Cost the same but is more productive.

How do I produce output y using only x_1? How much x_1 do I need?

$$2x_1 = y$$

$$x_1 = \frac{y}{2}$$

The cost of this is $c = \frac{y}{2}$

If I only use x_2

$$x_2 = y$$

Cost of this is $c = y$

It is cheaper to use x_1 than x_2. So the optimal input bundle is:

$$x_1^* = \frac{y}{2}, x_2^* = 0$$

These are called the **conditional factor demands**.

What is of cost of producing y in the cheapest way? We calculate how much the optimal bundle costs. Cost of the conditional factor demands:

$$w_1 x_1^* + w_2 x_2^*$$

$$c(y) = \frac{y}{2}$$

Firm's Cost function.

$$mc(y) = \frac{\partial (c(y))}{\partial y} = \frac{1}{2}$$

Constant marginal cost. *Every extra unit of output costs* $\frac{1}{2}$.

Other possibilities:

Increasing marignal cost. mc is increasing in y

Decreasing marginal cost. mc is decreasing in y

1.4.3 You Try #1

What are the conditional factor demands for producing output y. What is the cost function?

$w_1 = 1, w_2 = 1$

$f(x_1, x_2) = min\left\{\frac{1}{2}x_1, x_2\right\}$

Use the no-waste condition, plus the production constraint $min\left\{\frac{1}{2}x_1, x_2\right\} = y$ to find the conditional factor demand and the cost function.

No waste condition:

$$\frac{1}{2}x_1 = x_2$$

Production constaint:

$$min\left\{\frac{1}{2}x_1, x_2\right\} = y$$

Plug no-waste into the production constraint.

$$min\{x_2, x_2\} = y$$

Conditional factor demand for x_2

$$x_2 = y$$

Plug this into the no waste condition:

$$\frac{1}{2}x_1 = y$$

Conditional factor demand for x_1

$$x_1^* = 2y$$

Cost of this:

$$c(y) = 2y + y = 3y$$

Marginal cost:

$$mc(y) = 3$$

1.4.4 You Try #2

$w_1 = 1, w_2 = 1$

$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$

Tangency condition:

$$TRS = -\frac{w_1}{w_2}$$

$$-\frac{\frac{1}{2}x_1^{\frac{1}{2}-1}x_2^{\frac{1}{2}}}{x_1^{\frac{1}{2}}\frac{1}{2}x_2^{\frac{1}{2}-1}} = -\frac{1}{1}$$

$$-\frac{\frac{1}{2}x_1^{-\frac{1}{2}}x_2^{\frac{1}{2}}}{\frac{1}{2}x_1^{\frac{1}{2}}x_2^{-\frac{1}{2}}} = -1$$

$$-\frac{\frac{1}{2}x_2^{\frac{1}{2}}x_2^{\frac{1}{2}}}{\frac{1}{2}x_1^{\frac{1}{2}}x_1^{\frac{1}{2}}} = -1$$

$$\frac{x_2}{x_1} = 1$$

$$x_2 = x_1$$

Production constraint:

$$x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = y$$

Plug in the tangency condition $x_2 = x_1$

$$x_1^{\frac{1}{2}} x_1^{\frac{1}{2}} = y$$

Conditional factor demand for x_1

$$x_1^* = y$$

Conditional factor demand for x_2

$$x_2^* = y$$

Calculate the cost of the to get the cost function:

$$1y + 1y = 2y$$

$$c(y) = 2y$$

1.4.5 Short Run Cost

We say a company is in the "short-run" if one of the inputs is fixed.

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

Conditional factor demands: $x_1 = y, x_2 = y, c(y) = 2y$

In the short run, suppose $x_2 = 4$.